BEGN 122A Fall 2024

Quiz 2

Tuesday, November 19, 2024

***Name** (Last, First):* SOLUTIONS

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 21, 2024 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

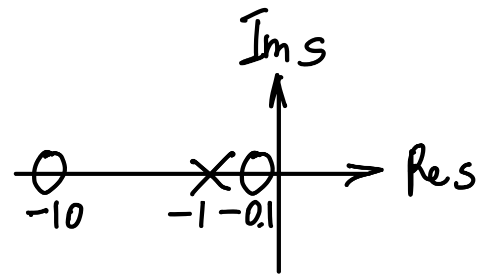
1. **[50 pts]** Consider the following linear time-invariant (LTI) biosystem:

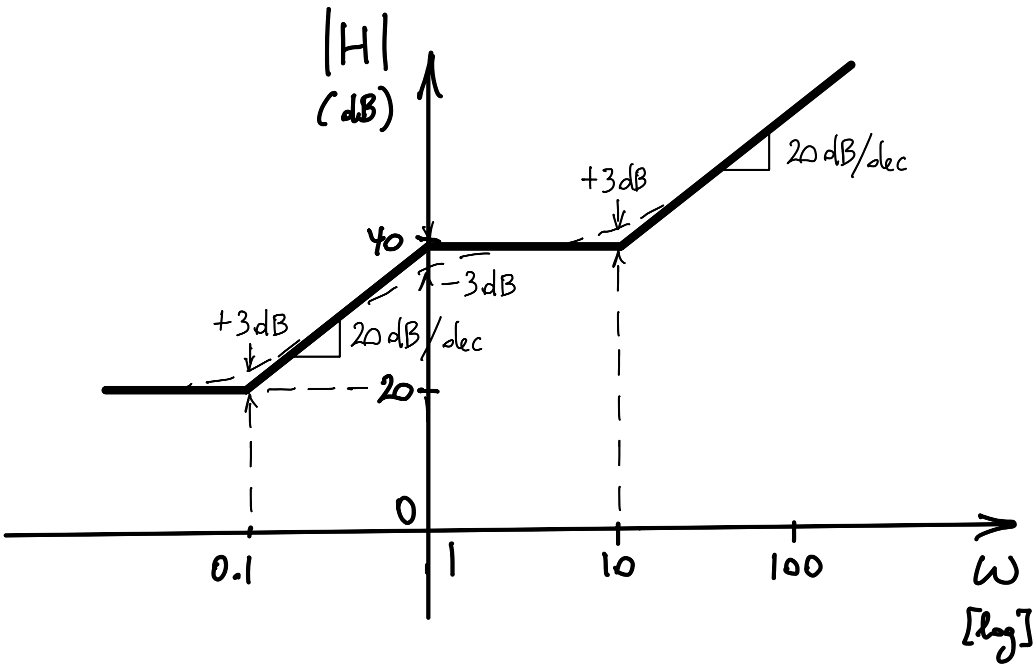
$$H(s) = \frac{10s^2 + 101s + 10}{s+1}. = \frac{(10s+1)(s+10)}{s+1}$$

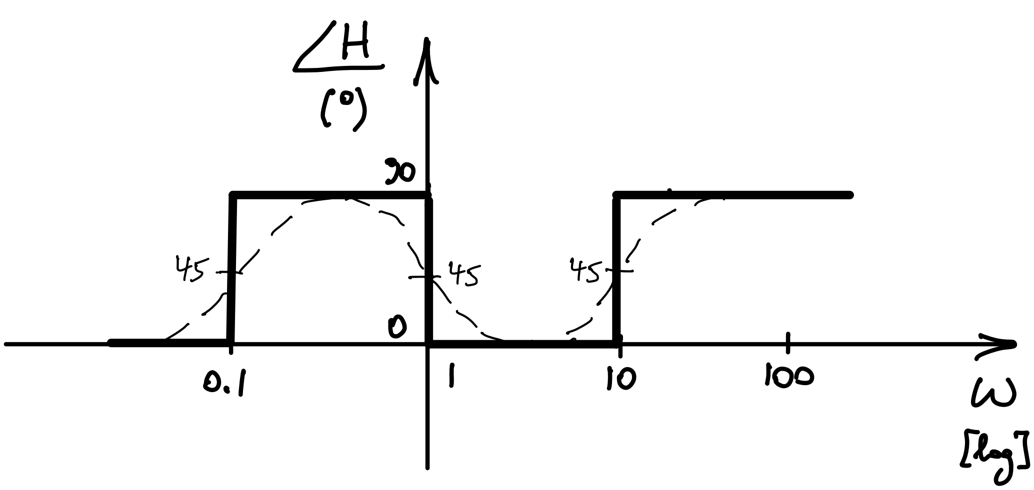
(a) [10 pts] Sketch the Bode plot.

Zeros @ $s = -0.1, -10$

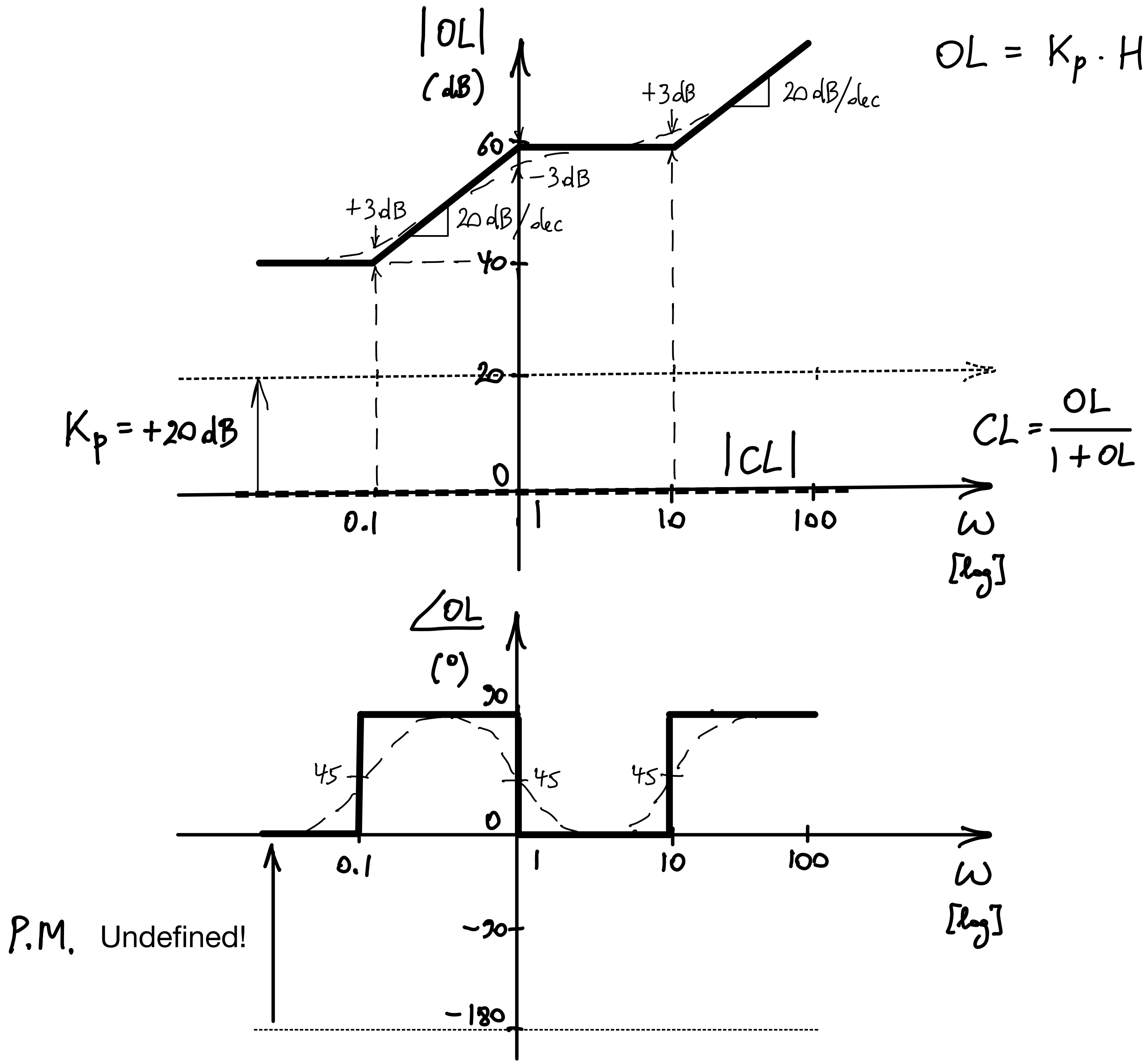
Pole @ $s = -1$







(b) [10 pts] First consider proportional control with 20 dB gain ($K_p = 10$), without measurement error. Find the DC error of the closed-loop system. What can you say about the phase margin, and the closed-loop bandwidth?



DC error $= 1\% = -40\,\text{dB}$ $\left(OL(0) = 100, \text{ so } CL(0) = \frac{100}{101} = 0.99 \right)$

Phase margin UNDEFINED (because nowhere $|OL| = 1$, but guaranteed stable)

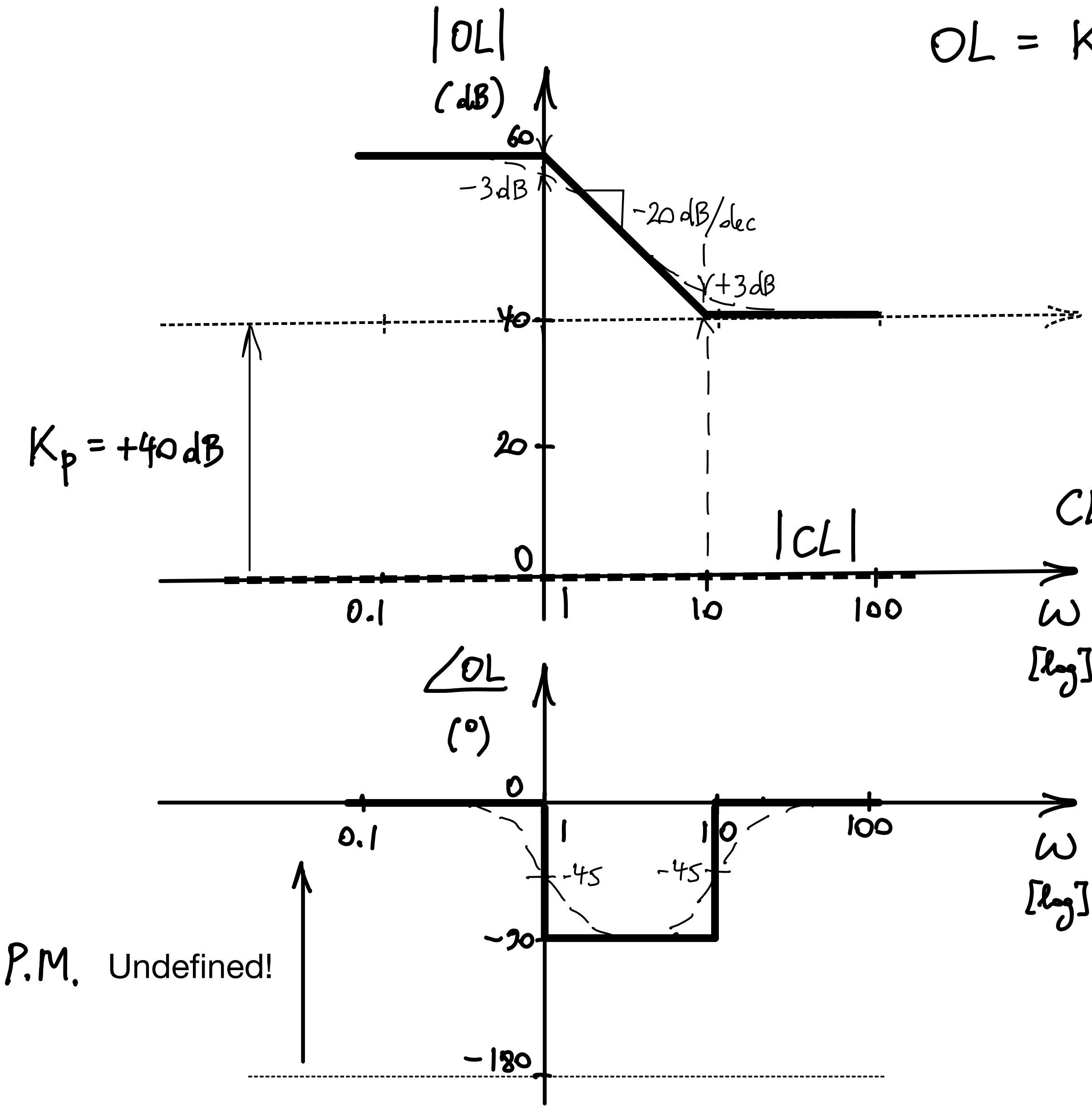
Closed-loop $-3\,\text{dB}$ BW $= \infty$ $\left(|CL| \cong 1 \text{ for all } \omega \right)$

(c) [10 pts] Now consider error in the measurement of the biosystem, with the measurement system given by:

$$G(s) = \frac{1}{10s + 1}.$$

Find the value of the proportional gain K_p to give no more than 0.1% DC error. Does this affect the phase margin? Explain.

$OL = K_p \cdot H \cdot G$



$CL = \frac{OL}{1 + OL}$

DC error = 0.1 % = −60 dB for K_p = 100 = +40 dB $\left(CL(0) = \frac{1000}{1001} = 0.999 \right)$

Phase margin remains UNDEFINED (and still guaranteed stable)

(d) [10 pts] Consider the biosystem again with the same measurement error and proportional control gain K_p, but now including integral control. Find the maximum value for the integral gain K_i to maintain stability. Find the DC error of the closed-loop system.

Infinite. Any positive value of the integral gain Ki maintains stability, because the open-loop gain magnitude |OL| remains larger than 1 (0 dB) for all frequencies, so that the phase margin remains undefined with guaranteed stability.

The DC error of the closed-loop system is 0 (i.e., CL(0) = 1) because the open-loop DC gain is infinite owing to the pole at zero by the integral control.

(e) [10 pts] Would additional derivative control help to improve system performance in any way? Explain.

Not really. Phase margin is already undefined guaranteeing stability, and closed-loop bandwidth is already infinite, with the closed-loop gain close to 1 for all frequencies. Any additional derivative control can only marginally improve the closed-loop system response at high frequencies, bringing the closed-loop gain even closer to 1.

2. **[30 pts]** Here we consider the dynamics of a coupled set of two ordinary differential equations in state variables $u(t)$ and $v(t)$ describing the biomechanics of a biosystem driven by a force input $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -\gamma\, v(t) + f(t) \end{aligned}$$

with positive mass m, but with negative damping $\gamma < 0$.

(a) [10 pts] Find the transfer function $H(s) = u(s)\,/\,f(s)$ of the biosystem, and find the poles and zeros. Is the biosystem stable? Explain.

$$s\, u(s) = v(s)$$

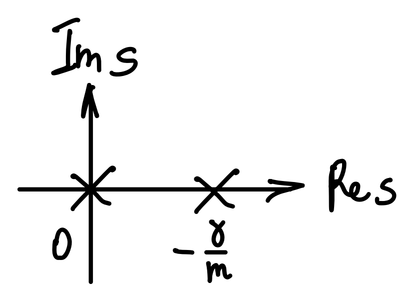
$$m s\, v(s) = -\gamma\, v(s) + f(s)$$

$$\Rightarrow \quad m s^2 u(s) = -\gamma s\, u(s) + f(s)$$

$$H(s) = \frac{u(s)}{f(s)} = \frac{1}{m s^2 + \gamma s} = \frac{1}{s(ms+\gamma)}$$

Two poles @ $s = 0, -\frac{\gamma}{m}$

No zeros



UNSTABLE for $\gamma < 0$

(b) [10 pts] Consider derivative control $F(s) = K_d\, s$ acting on the biosystem without measurement error. For what range of values of K_d is the closed-loop system stable? Explain.

$$OL(s) = F(s) \cdot H(s) = \frac{K_d \cancel{s}}{\cancel{s}(ms+\gamma)} = \frac{K_d}{ms+\gamma}$$

$$CL(s) = \frac{OL(s)}{1+OL(s)} = \frac{K_d}{ms+\gamma+K_d}$$

Single pole @ $s = -\frac{\gamma+K_d}{m}$

STABLE for $\gamma + K_d \geq 0$

$\Rightarrow K_d \geq -\gamma$

(Range : $[-\gamma, +\infty]$)

(c) [10 pts] Now consider the effect of measurement error acting on the biosystem with additional gain

$$G(s) = \frac{1}{1+\tau s}$$

in the open-loop transfer function. Find the value of the derivative control gain K_d for which the closed-loop system is stable and critically damped.

$$OL(s) = F(s) \cdot H(s) \cdot G(s) = \frac{K_d}{(ms+\gamma)(1+\tau s)}$$

$$CL(s) = \frac{OL(s)}{1+OL(s)} = \frac{K_d}{(ms+\gamma)(1+\tau s) + K_d}$$

$$= \frac{K_d}{m\tau s^2 + (m+\gamma\tau)s + \gamma + K_d}$$

Critically damped for:

$$(m+\gamma\tau)^2 - 4m\tau(\gamma+K_d) = 0$$

$$\Rightarrow K_d = -\gamma + \frac{(m+\gamma\tau)^2}{4m\tau}$$

Stability also requires:

$$m\tau \geq 0 \quad (\text{ok})$$

$$m + \gamma\tau \geq 0$$

$$\gamma + K_d \geq 0 \quad (\text{ok})$$

3. **[20 pts]** Short answer questions– let your imagination and creativity go loose!

(a) [10 pts] Give an example of a biosystem where integral control substantially improves the closed-loop DC error. Explain.

Any biosystem with a zero at zero. Its open-loop DC gain is zero so its closed-loop DC gain is zero giving 100% error. Integral control gives non-zero closed-loop gain substantially reducing the closed-loop DC error.

For instance, velocity of a wheelchair driven by a motor. Integral control settles to the target velocity by integrating the velocity error for the motor to rotate the wheels.

(b) [10 pts] Give an example of a biosystem where derivative control substantially improves the high-frequency response. Explain.

Any biosystem with near-zero phase margin. Derivative control lifts the phase and boosts the gain at high frequencies. Chosen properly, this increases the phase margin by between 45 and 90 degrees for much improved stability. The increased open-loop gain also increases the closed-loop bandwidth.

For instance, the biomechanics of an artificial limb that is severely underdamped. Derivative control in the prosthesis adds damping to improve its stability. Critical damping by tuning the derivative gain gives fast settling without ringing in the transient dynamics.