1. [30 pts] A Severinghaus electrode, shown below, is used to measure partial pressure of carbon dioxide (PCO$_2$) in a blood sample at room temperature. The internal solution of the glass electrode is 10 mmol/l of HCl. The voltage on the glass electrode relative to the reference electrode is measured with a voltmeter, reading -85 mV. The experiment is repeated for a calibration sample with known PCO$_2$ of 128 mmHg, now reading a voltage of -52 mV.

(a) [IC] [5 pts] What is the preferred type of reference electrode for this instrument? Explain your choice.

(b) [10 pts] Find the pH of the buffer solution surrounding the reference electrode, for the blood sample, and for the calibration sample.

(c) [10 pts] Find PCO$_2$ for the blood sample.

(d) [5 pts] The flow rate of the blood sample through the chamber is now increased. Would you expect the voltmeter reader to increase, decrease, or stay the same? Explain.
2. [20 pts] A protein called Martian blood protein (Mb) has been identified in Martian blood. Mb binds 
CO$_2$, and the CO$_2$ saturation of Mb in Martian blood is of interest to researchers. Unsaturated Mb 
absorbs violet light and allows green light to pass, while saturated Mb (MbCO$_2$) absorbs green light 
and allows violet light to pass. The absorption spectra of Mb and MbCO$_2$ are shown below. Note 
the logarithmic scale on the $y$ axis!

![Absorption Spectra](image)

(a) [IC] [5 pts] What is the isosbestic wavelength $\lambda_i$ for Mb? What is the absorptivity $\alpha_i$ at this 
wavelength?

(b) [15 pts] If you were designing a Martian carbon-dioximeter with 2 light sources, what wave-
lengths, $\lambda_1$ and $\lambda_2$, would you pick for your light sources? Find an expression for CO$_2$ satu-
ration (SCO$_2$) in terms of the absorbances at your chosen wavelengths, $A(\lambda_1)$ and $A(\lambda_2)$. 

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3. **20 pts** Consider a wearable sensor that uses ISFETs (ion-sensitive field effect transistors) for monitoring electrolytes in sweat to alert and prevent dehydration during physical exercise. The sensor is mounted on the user’s arm, with the ISFET gate in contact with the skin. The ISFET interfaces with an active circuit shown below to measure the concentration of targeted ions \(S^+\) in the sweat.

\[ V_S = 100 \text{ mV}, \ R_i = 50 \ \Omega, \ R_a = 1 \text{ M} \Omega, \ R_b = 1 \text{ k} \Omega, \text{ and } R_c = 100 \text{ k} \Omega. \]

The conductance \(g_{ds}\) of the ISFET is approximately proportional to the ion concentration: \[ g_{ds} = \frac{W}{L} \mu k[S^+] \]

with width \(W = 100 \mu m\), length \(L = 20 \mu m\), electron mobility \(\mu = 0.12 \text{ m}^2/\text{Vs}\), and charge-coupling factor \(k = 0.1 \text{ CL/m}^2\text{mol}\).

![ISFET Circuit Diagram]

(a) **10 pts** Derive \(V_o\) as a function of the target ion concentration \([S^+]\), and write the sensitivity of the sensor. What is the smallest concentration that you can detect if the output \(V_o\) is quantized with 100 \(\mu\)V LSB resolution?

(b) **10 pts** What purpose does \(R_i\) serve in protecting the health of the user and the circuit from accidental electrical shorting of the ISFET? Explain.

4. **Design Problem [30 pts]**: Design a pulse oximeter system using a 3 V battery, a single red LED, a single infrared LED, and a single photodiode that is equally sensitive to red and infrared light. The red and infrared LEDs need to be activated at 500 Hz pulse rate in alternating fashion (1 ms red followed by 1 ms infrared, repeated at 500 Hz rate). Your system should output two voltage signals, one directly proportional to incident red light intensity and the other directly proportional to incident infrared light intensity on the photodiode. You may use as many resistors, capacitors, opamps, timers, switches, and gates as you need. Protect your circuit from accidental shorting of the voltage output so that no more than 10 \(\mu\)A may be drawn from the output.

*Hint: In the reverse bias mode, a photodiode outputs a current that is linearly proportional to light intensity.*