BENG 186B Winter 2019 Final

Thursday, March 21, 2019

Last Name, First Name: SOLUTIONS

- This final is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- This final has 21 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 10 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- You have 3 hours to complete this final.

1	/16
2	/14
3	/16
4	/8
5	/6
6	/8
7	/10
8	/8
9	/10
10	/4
Total	**/100**

You may find the following equations useful:

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad \zeta = \frac{1}{2} RC\,\omega_n$$

$$R = R_G\,(1 + G\,\epsilon) \qquad \sigma = E\,\epsilon$$

$$V_o = A_d V_d + A_c V_{cm}$$

$$V_d = V_b - V_a$$

$$V_{cm} = (V_b + V_a)/2$$

$$V = \mathbf{M} \cdot \mathbf{r} = \cos\theta\,|\mathbf{M}|\,|\mathbf{r}|$$

$$e = \int_0^\ell \mathbf{v} \times \mathbf{B}\,d\ell$$

$$\Delta f = \frac{v}{c}(\cos\theta_r + \cos\theta_s) f_s$$

$$V = E_{\text{glass}} - E_{\text{ref}} + E_{\text{Nernst}}$$

$$E_{\text{Nernst}} = \frac{RT}{nF}\ln(10)\log_{10}\left(\frac{[\mathrm{A^n}]_{\text{out}}}{[\mathrm{A^n}]_{\text{in}}}\right) \quad \text{for some ion } \mathrm{A^n} \text{ with valence } n$$

$$V_m = \frac{RT}{F}\ln(10)\log_{10}\frac{P_{\text{Na}}[\mathrm{Na^+}]_o + P_{\text{K}}[\mathrm{K^+}]_o + P_{\text{Cl}}[\mathrm{Cl^-}]_i}{P_{\text{Na}}[\mathrm{Na^+}]_i + P_{\text{K}}[\mathrm{K^+}]_i + P_{\text{Cl}}[\mathrm{Cl^-}]_o}$$

$$\frac{RT}{F}\ln(10) = 62\,\text{mV at room temperature}$$

$$I = 4F\,[\mathrm{O_2}]\,\phi \qquad F = 96\,485\,\mathrm{C/mol}$$

$$\log_{10}\mathrm{PCO_2} = -\mathrm{pH} + \text{constant}$$

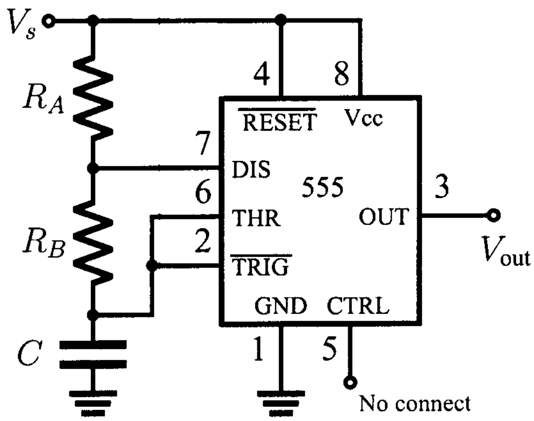
$$\mathrm{O_2 + 2\,H_2O + 4\,e^- \rightleftharpoons 4\,OH^-}$$

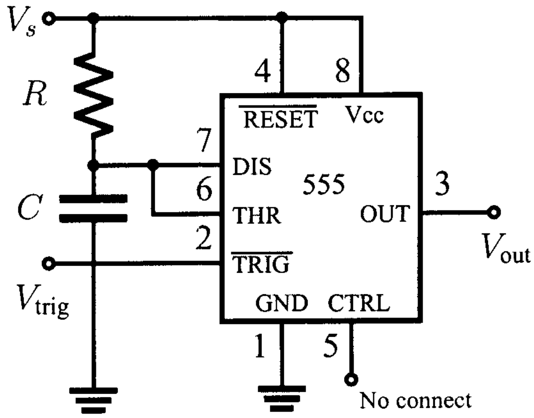
$$\mathrm{4\,Ag + 4\,Cl^- \rightleftharpoons 4\,AgCl + 4\,e^-}$$

$$A(\lambda) = W\,L\,a(\lambda)$$

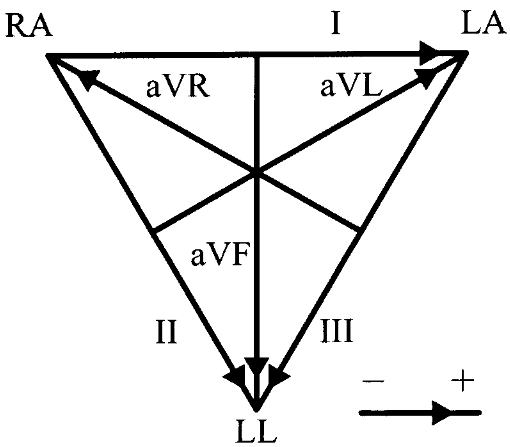
$$I_d = \frac{1}{1 - \exp(-d/\tau)}\, I_r$$

The following diagrams may come handy as well:





$T_{\text{hi}} = 0.7(R_A + R_B)C \qquad T_{\text{lo}} = 0.7R_BC \qquad\qquad T = 1.1RC$



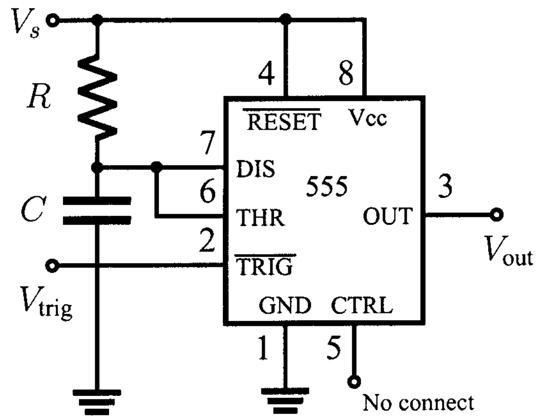
1. **[16 pts]** Circle the **best answer (only one answer per question)**:

   (a) [1 pt] Noise in a measurement system affects its:

      i. sensitivity
      ii. accuracy
      iii. precision
      iv. resolution
      v. none of the above

   (b) [1 pt] The circuit shown below outputs a high pulse at V_{out} when:

      i. CTRL is activated
      ii. THR goes above 2/3 V_S
      iii. V_{trig} goes above 2/3 V_S
      iv. V_{trig} goes below 1/3 V_S
      v. None of the above



   (c) [1 pt] A cell membrane permeable to only one ion type has an equilibrium potential equal to its:

      i. half-cell potential
      ii. Nernst potential
      iii. action potential
      iv. concentration overpotential
      v. All of the above

   (d) [1 pt] Which of the following bioptentials can be observed on electrodes placed over the skull:

      i. EEG
      ii. EMG
      iii. EOG
      iv. ECG
      v. All of the above

(e) [1 pt] The concentration of carbon dioxide in the blood can be derived from measurement of:

i. pH and $[HCO_3^-]$
ii. PO_2
iii. SO_2, pH, and temperature
iv. SCO_2
v. All of the above.

(f) [1 pt] The Severinghaus electrode:

i. Measures saturation of carbon dioxide in the blood.
ii. Implements driven right leg active grounding.
iii. Is an amperometric sensor requiring a transimpedance amplifier
iv. Measures oxygen concentration in the blood at constant flow rate.
v. None of the above.

(g) [1 pt] Which one of these devices provides non-invasive measurement of blood pressure?

i. A micro–tipped manometer.
ii. A fluid–filled catheter with pressure transducer.
iii. A sphygmomanometer.
iv. A tonometer.
v. All of the above.

(h) [1 pt] Microshock:

i. Is naturally less severe than macroshock.
ii. Has low rates of mortality when properly treated immediately.
iii. Is the state of disbelief when going through these kinds of questions.
iv. Is a major cause of cardiac arrest.
v. None of the above.

(i) [8 pts] Indicate for each statement below whether it is true or false:

i. **TRUE** / **FALSE**: A piezoelectric crystal transduces between stress, strain, charge, and voltage.

ii. **TRUE** / **FALSE**: An ion-sensitive field-effect transistor transduces concentration of an electrically charged chemical compound in solution to an electrical conductance.

iii. **TRUE** / **FALSE**: The current dipole generates an electric potential proportional to polarized current and inversely proportional to distance.

iv. **TRUE** / **FALSE**: The Ag/AgCl electrode is a non-polarizable electrode.

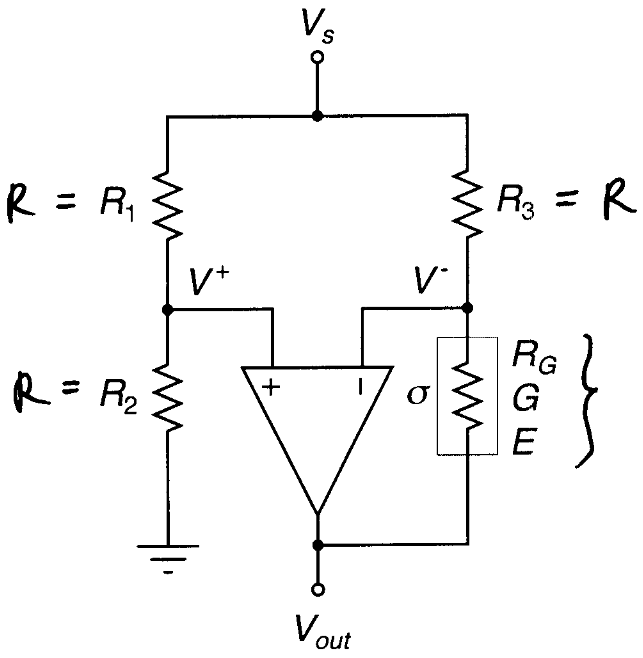
v. **TRUE** / **FALSE**: The P wave in the electrocardiogram manifests atrial depolarization.

vi. **TRUE** / **FALSE**: An underdamped second-order system responds with large transient oscillations to sharp changes in the input.

vii. **TRUE** / **FALSE**: An ideal transformer transfers electrical energy from input to output with minimal energy losses, and with galvanic isolation between input and output.

viii. **TRUE** / **FALSE**: Capacitive sensors are highly sensitive to displacement of the electrical plates.

2. **[14 pts]** Consider the active circuit below with a strain gauge in a modified Wheatstone bridge. The circuit is used as a transducer to measure stress σ by an output voltage V_{out}. The strain gauge has Young's modulus E, gauge factor G, and nominal resistance $R_G = R$. The other resistances R_1, R_2 and R_3 are constant and all also equal to R so that the bridge is perfectly balanced. You may assume the opamp is ideal.



$$R_G(1 + G\varepsilon) = R\left(1 + \frac{G}{E}\sigma\right)$$

(a) [2 pts] Find the node voltages V^+ and V^- for a fixed source voltage V_s.

$$R_1 = R_2 = R \Rightarrow V^+ = V^- = \frac{V_s}{2}$$

(b) [6 pts] Find the output voltage V_{out}.

$$V_{out} = V^- - R\left(1 + \frac{G}{E}\sigma\right) \cdot \frac{V_s - V^-}{R}$$

$$= \cancel{\frac{V_s}{2}} - \left(\cancel{1} + \frac{G}{E}\sigma\right)\frac{V_s}{2} = -\frac{1}{2}\frac{G}{E}V_s \cdot \sigma$$

(c) [4 pt] Find the sensitivity. Does it depend on the stress σ, and why?

$$S = \frac{dV_{out}}{d\sigma} = -\frac{1}{2}\frac{G}{E}V_s \quad \text{independent of } \sigma$$

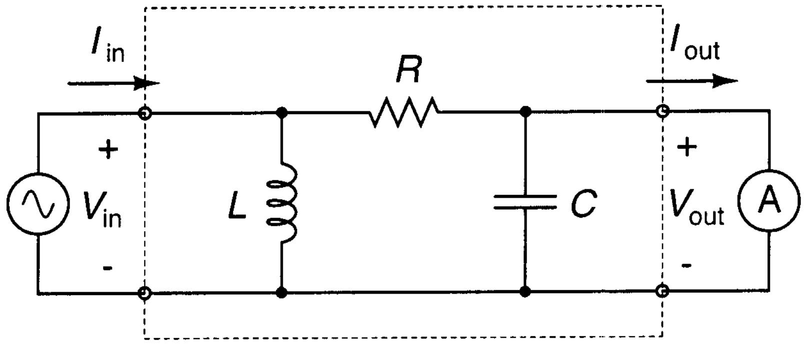
The active circuit linearly transduces strain gauge resistance into output voltage.

(d) [2 pt] Find the output impedance on the V_{out} node.

$$Z_{out} = \frac{\Delta V_{out}}{\Delta I_{out}} = 0$$

: V_{out} is independent of current drawn from the opamp output.

3. **[16 pts]** Now analyze the following voltage-in, current-out filter circuit:



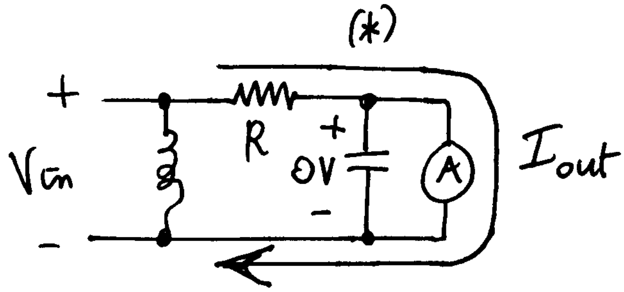
(a) [1 pts] What is the impedance of the ideal voltage source driving the input of the filter?

$0\ \Omega$

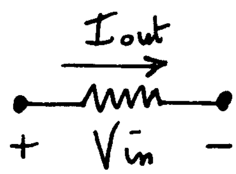
(b) [1 pts] What is the impedance of the ideal current meter loading the output of the filter?

$0\ \Omega$

(c) [3 pt] Find the transfer function $H(j\omega) = I_{out}(j\omega)/V_{in}(j\omega)$. Does it depend on frequency?



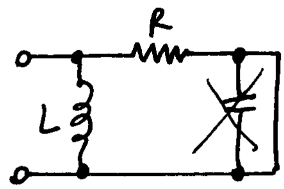
or:



$$\Rightarrow H(j\omega) = \frac{I_{out}}{V_{in}} = \frac{1}{R}$$ independent of frequency.

(*) Zero volts across the capacitor $\Rightarrow$ zero current (charge) through the capacitor.

(d) [3 pts] Find the input impedance $Z_{in}(j\omega)$ of the filter.

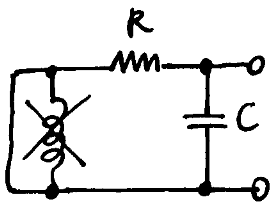


Z_{in}

$$Z_{in}(j\omega) = j\omega L \,\|\, R = \frac{j\omega L R}{j\omega L + R} = \frac{j\omega L}{1 + j\omega \frac{L}{R}}$$

(HIGHPASS)

(e) [2 pts] Find the output impedance $Z_{out}(j\omega)$ of the filter.



Z_{out}

$$Z_{out}(j\omega) = \frac{1}{j\omega C} \,\|\, R = \frac{\frac{1}{j\omega C} \cdot R}{\frac{1}{j\omega C} + R} = \frac{R}{1 + j\omega RC}$$

(LOWPASS)

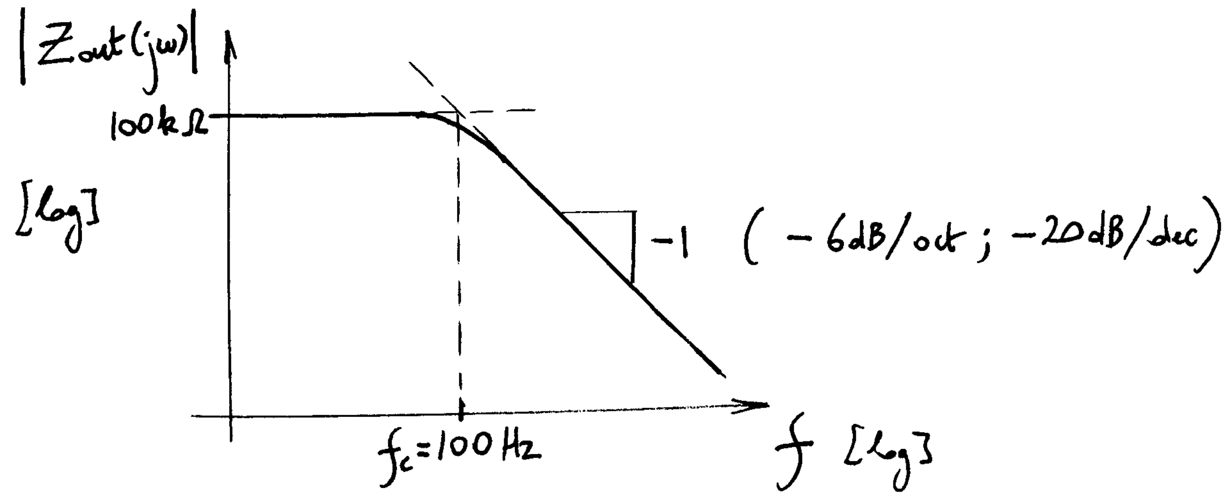
(f) [6 pt] Sketch the Bode plot of the output impedance $Z_{out}(j\omega)$ from 0.1 Hz to 10 kHz, for $R = 100\ \text{k}\Omega$, $L = 23$ nH, and $C = 16$ nF. Be sure to indicate all units.

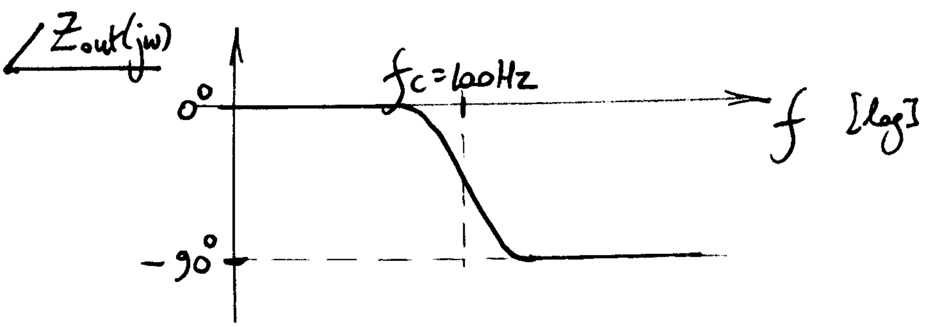
$$Z_{out}(j\omega) = \frac{R}{1 + j\omega RC} = \frac{R}{1 + j\frac{\omega}{\omega_c}}$$

with cut-off $\omega_c = \dfrac{1}{RC}$

$$f_c = \frac{1}{2\pi}\frac{1}{RC} = \frac{1}{6.28}\frac{1}{10^5\,1.6\,10^{-8}}\ \text{Hz}$$

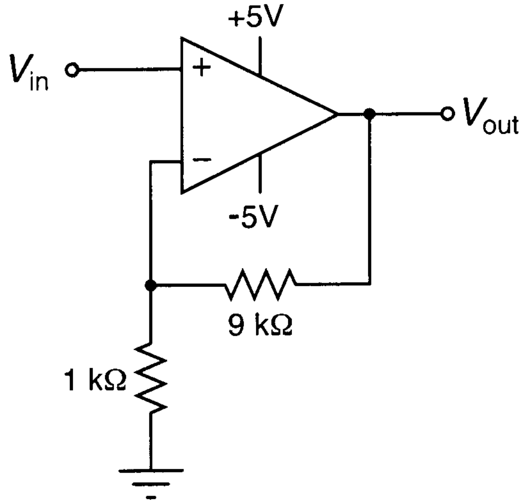
$$= 100\ \text{Hz}$$

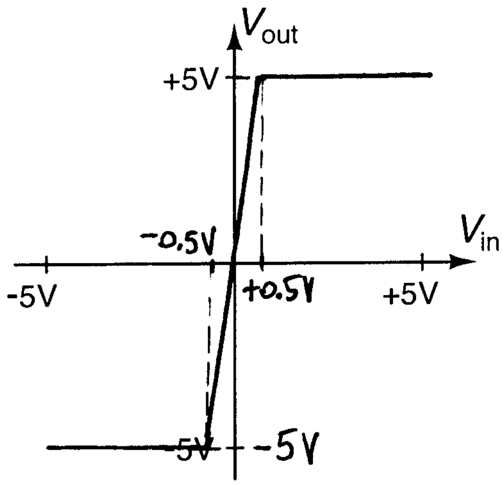




4. **[8 pts]** For each of the two different active circuits below, sketch V_{out} versus V_{in} on the diagram on the right. Show your reasoning.

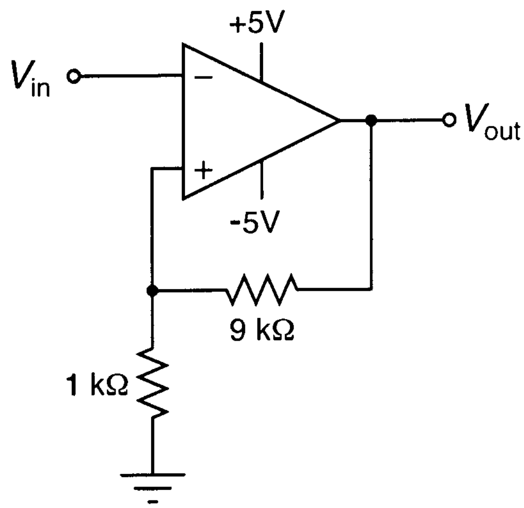
(a) [4 pts]

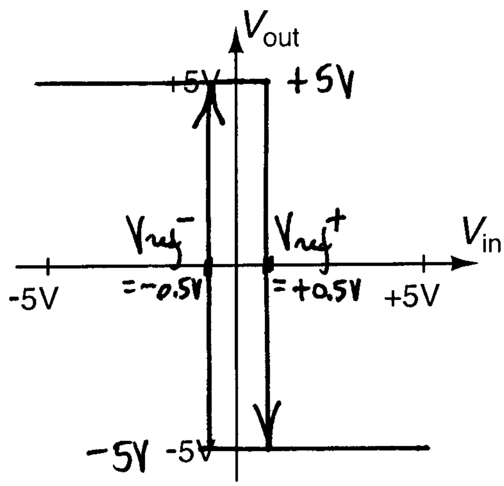




NON-INVERTING AMPLIFIER with gain: $A = 1 + \frac{9k\Omega}{1k\Omega} = 10$

(b) [4 pt]

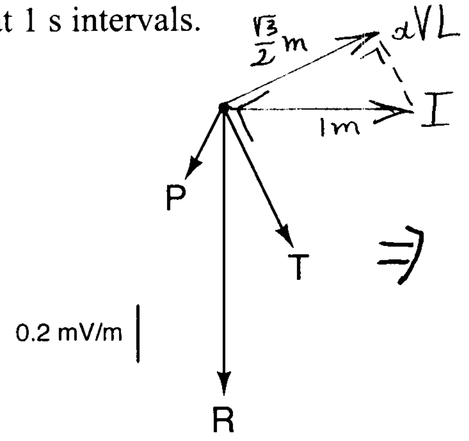




INVERTING HYSTERETIC COMPARATOR with hysteretic levels:

$$\begin{cases} V_{ref}^+ = \dfrac{1k\Omega}{1k\Omega + 9k\Omega} \cdot 5V = +0.5V \\ V_{ref}^- = \quad " \quad \cdot (-5V) = -0.5V \end{cases}$$

5. **[6 pts]** The cardiac vector **M** in the frontal plane points vertically downwards with amplitude 1 mV/m during the R wave of the ECG, shifts 30 degrees sideways pointing towards the left leg with amplitude 0.5 mV/m during the T wave, and -30 degrees sideways pointing towards the right leg with amplitude 0.2 mV/m during the P wave, as shown below. The distance between the patient's shoulders is 1 m, and the patient's heart beats at 1 s intervals.



$\Rightarrow$ P $\parallel$ III and P $\perp$ aVR

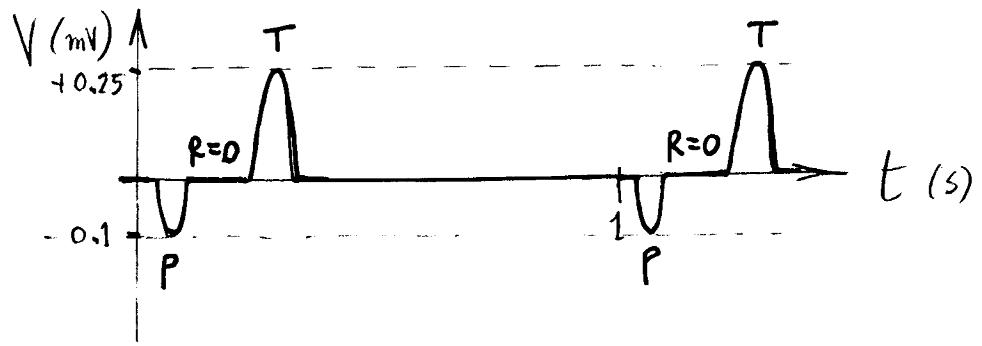
R $\parallel$ aVF and R $\perp$ I

T $\parallel$ II and T $\perp$ aVL

(a) [3 pts] Sketch one cycle of the ECG voltage as a function of time, recorded along lead I. Indicate units on your axes.

$$T: 0.5\,\frac{mV}{m} \cdot 1m \cdot \underbrace{\cos(60^\circ)}_{=\frac{1}{2}} = 0.25\ mV$$

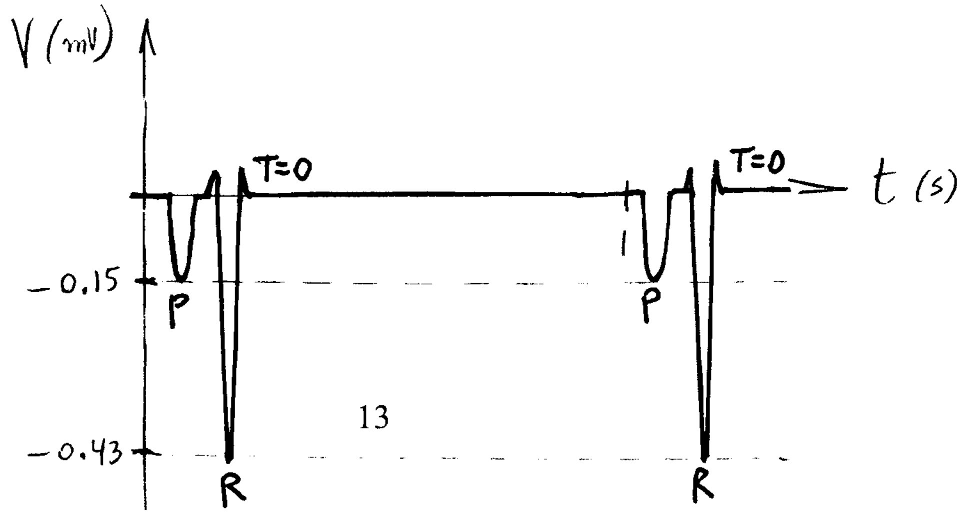
$$P: 0.2\,\frac{mV}{m} \cdot 1m \cdot \underbrace{\cos(120^\circ)}_{=-\frac{1}{2}} = -0.1\ mV$$



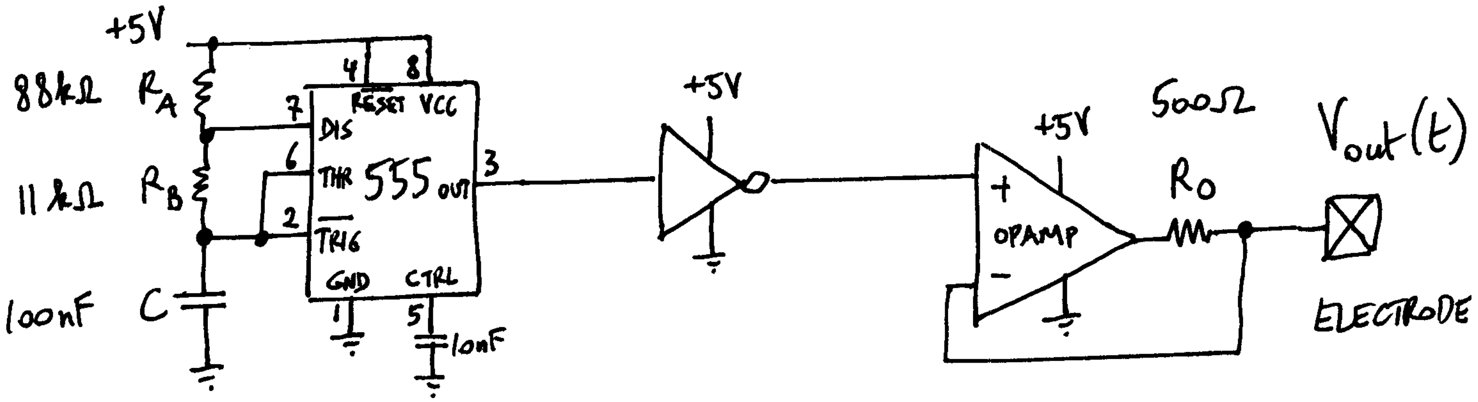
(b) [3 pts] Again, sketch one cycle of the ECG waveform, but now recorded along lead aVL.

$$P: 0.2\,\frac{mV}{m} \cdot \frac{\sqrt{3}}{2}m \cdot \underbrace{\cos(150^\circ)}_{=-\frac{\sqrt{3}}{2}} = -0.15\ mV$$

$$R: 1\,\frac{mV}{m} \cdot \frac{\sqrt{3}}{2}m \cdot \cos(120^\circ) = -0.43\ mV$$



6. **[8 pts]** Design a simple deep brain stimulator (DBS) that continuously generates a 130 Hz periodic square wave voltage output signal $V_{out}(t)$ with pulse height 5 V and duty cycle 0.1 (*i.e.*, the output should be 0 V 90% of the time, and 5 V 10% of the time, alternating between these two twice every period). Include also short-circuit protection at the voltage output, drawing no more than 10 mA from an accidental short to ground. You may use any active devices that you learned in class, any combination of resistors and capacitors, and a single 5 V battery. You may also assume that all active devices include protection diodes to the power supplies at their input and output terminals.



555 TIMER ASTABLE

5V pulse

130 Hz frequency

0.9 duty cycle

(always > 0.5 for the 555)

LOGIC INVERTER

5V pulse

130 Hz frequency

0.1 duty cycle

CURRENT LIMITING VOLTAGE BUFFER

5V pulse

130Hz frequency

0.1 duty cycle

10mA short circuit

$$T_{lo} = 0.7\, R_B C = \frac{1}{10}\,\frac{1}{130}\ \text{s}$$

e.g. $C = 100\text{nF}$; $R_B = 11\text{k}\Omega$

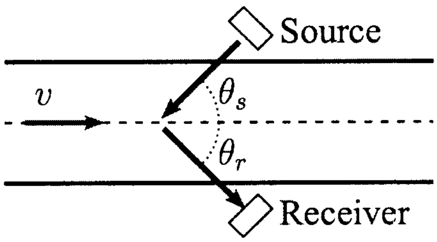
$$T_{hi} = 0.7\,(R_A + R_B)\,C = 9\,T_{lo} \Rightarrow R_A = 8R_B = 88\text{k}\Omega$$

$$\frac{5\text{V}}{R_o} = 10\text{mA}$$

$$\Rightarrow R_o = \frac{5\text{V}}{10\text{mA}} = 500\Omega.$$

7. **[10 pts]** Physiological signals as indicators of cardiovascular health are monitored using a variety of bioinstruments, that can be entirely non-invasive. Here we consider two such signals of interest: blood velocity, and oxygen saturation.

   (a) [4 pts] Blood velocity is measured non-invasively using a Doppler transducer, as shown below. Find the transmit and receive angles θ_s and θ_r that maximize the sensitivity. Are these angles practical? What angles are usually adopted in practice? Explain.



$$\Delta f = \frac{v}{c}(\cos\vartheta_r + \cos\vartheta_s)\cdot f_s$$

Sensitivity: $S = \dfrac{d(\Delta f)}{dv} = \dfrac{f_s}{c}\cdot(\cos\vartheta_r + \cos\vartheta_s)$

maximized for $\cos\theta_r = \cos\vartheta_s = +1$ (or -1 in amplitude)

$\vartheta_r = \vartheta_s = 0^\circ$ (or 180°)

This is not practical for a NON-INVASIVE sensor.

A practical compromise: $\vartheta_r = \vartheta_s = \pm 45^\circ$ (or $\pm 135^\circ$)

at a 30% loss in sensitivity

$\left(1 - \frac{\sqrt{2}}{2}\right)$

(b) [6 pts] A pulse oximeter is used to measure oxygen saturation in the blood non-invasively by monitoring absorbances at different optical wavelenghts. Given spectral measurements of the absorptivities $a_o(\lambda)$ and $a_r(\lambda)$ of oxygenated and reduced hemoglobin at two distinct wavelengths λ_1 and λ_2, find a general expression for the ratio of absorbances measured in the vessel at these two wavelengths, $A(\lambda_1)\,/\,A(\lambda_2)$, as a function of SO_2. Under what conditions is the relationship linear, and how do you maximize sensitivity?

Beer's law: $A(\lambda) = W \cdot L \cdot a(\lambda)$ for each compound

$$= (W \cdot SO_2) \cdot L \cdot a_o(\lambda) + (W \cdot (1-SO_2)) \cdot L \cdot a_r(\lambda)$$

$$\Rightarrow A(\lambda_1) = WL\left(SO_2\, a_o(\lambda_1) + (1-SO_2)\, a_r(\lambda_1)\right)$$

$$A(\lambda_2) = WL\left(SO_2\, a_o(\lambda_2) + (1-SO_2)\, a_r(\lambda_2)\right)$$

Ratiometric technique:

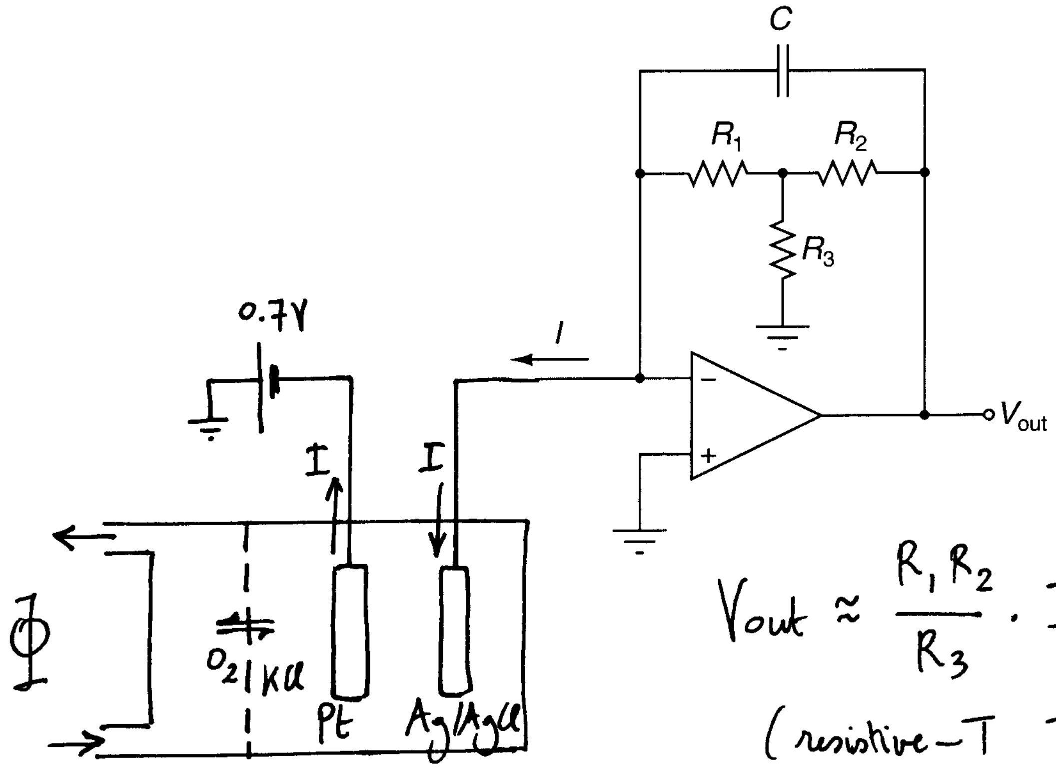
$$\frac{A(\lambda_1)}{A(\lambda_2)} = \frac{a_r(\lambda_1) + (a_o(\lambda_1) - a_r(\lambda_1))\, SO_2}{a_r(\lambda_2) + (a_o(\lambda_2) - a_r(\lambda_2))\, SO_2}$$

Linear when $a_o(\lambda_2) = a_r(\lambda_2)$ (ISOSBESTIC wavelength)

Sensitivity (slope) $\dfrac{a_o(\lambda_1) - a_r(\lambda_1)}{a_r(\lambda_2)}$ maximum for λ_1 where $|a_o(\lambda_1) - a_r(\lambda_1)|$ is largest

8. **[8 pts]** A Clark electrode is used to measure concentration of oxygen in the blood stream. The Clark "electrode" actually contains two electrodes: one Pt electrode, and one Ag/AgCl electrode, that are both immersed in the reference chamber of the Clark chemical cell. The reference chamber contains a saturated solution of KCl, and is separated from the sample chamber by an oxygen-permeable membrane. Blood passes through the sample chamber at a constant rate.

   (a) [4 pts] Complete the diagram below to realize an oxygen-to-voltage transducer that outputs a voltage V_{out} proportional to oxygen concentration $[O_2]$ in the blood. In addition to the transimpedance amplifier (TIA) circuit shown, you are given a Clark chemical cell with Pt and Ag/AgCl electrode terminals, and with blood passing through at known flow rate Φ. You are also given a 0.7 V battery, in addition to the ± 3 V power supplies to the TIA.



$$V_{out} \approx \frac{R_1 R_2}{R_3} \cdot I$$

(resistive-T TIA)

$$I = 4 F \Phi \cdot [O_2]$$

(Clark)

(b) [3 pts] Find the sensitivity of the oxygen-to-voltage transducer for $R_1 = R_2 = 1\,\mathrm{M\Omega}$ and $R_3 = 1\,\mathrm{k\Omega}$.

$$V_{out} = \frac{R_1 R_2}{R_3} 4F\Phi \cdot [O_2]$$

$$S = \frac{dV_{out}}{d[O_2]} = \frac{R_1 R_2}{R_3} \cdot 4F\Phi$$

$$= \underbrace{10^9\,\Omega}_{= \frac{Vs}{C}} \cdot 4 \cdot 96{,}485\,\frac{C}{mol} \cdot \Phi$$

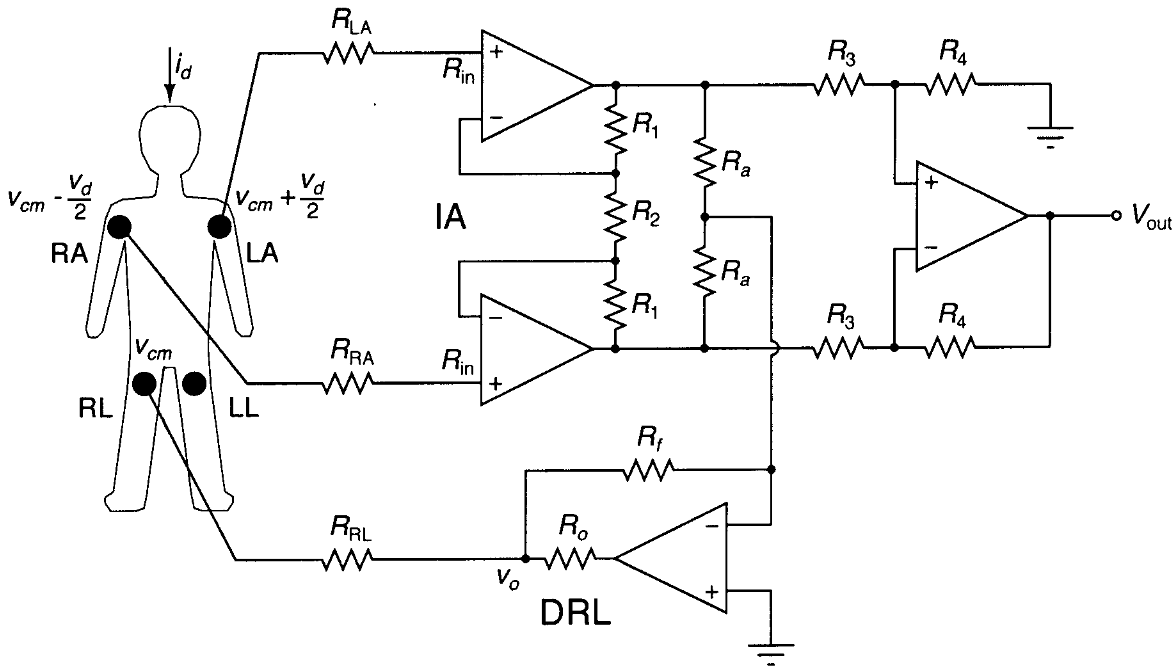
$$= 3.86\ 10^{14}\ \frac{Vs}{mol} \cdot \Phi$$

(c) [1 pts] Find the value of C in order to filter out noise contaminating the output V_{out} at frequencies 1 kHz and higher.

TIA effective transimpedance: $\dfrac{R_{eff}}{1 + j\omega R_{eff}.C}$ with $R_{eff} = 1G\Omega$

$$f_c = \frac{1}{2\pi R_{eff} C} = 1\,kHz \Rightarrow C = \frac{1}{2\pi\ 1G\Omega\ 1kHz} = 159\,fF$$

9. **[10 pts]** Consider the instrumentation amplifier (IA) and driven right leg (DRL) system below with resistances $R_1 = R_f = 495\,\text{k}\Omega$, $R_2 = R_3 = R_a = 10\,\text{k}\Omega$, $R_4 = 100\,\text{k}\Omega$, and with electrode resistances $R_{RA} = 110\,\text{k}\Omega$, $R_{LA} = 90\,\text{k}\Omega$, and $R_{RL} = 100\,\text{k}\Omega$. All opamp input impedances are $R_{in} = 1\,\text{G}\Omega$; otherwise, you may assume all active and passive components are ideal.



(a) [3 pts] Which lead of the ECG does this IA measure? Find the differential gain A_d, and effective common-mode rejection ratio CMRR_{eff}.

Lead I

$$A_d = \left(1 + 2\frac{R_1}{R_2}\right)\frac{R_4}{R_3} = 100 \cdot 10 = 1{,}000$$

$$\text{CMRR}_{eff} = \frac{R_{in}}{|R_{LA} - R_{RA}|} = \frac{1\,\text{G}\Omega}{20\,\text{k}\Omega} = 50{,}000$$

(b) [5 pts] For a peak current $i_d = 10\,\mu A$ entering the body, find the peak common-mode voltage v_{cm}, and the corresponding peak common-mode voltage error at the IA output V_{out}.

$$\text{DRL}: \quad v_{cm} = v_o + R_{RL} \cdot i_d \quad \text{and} \quad v_o = -\frac{2R_f}{R_d} v_{cm}$$

$$\Rightarrow v_{cm} = \frac{R_{RL}\, i_d}{1 + 2\frac{R_f}{R_d}} = \frac{100\,k\Omega \cdot 10\,\mu A}{1 + 99} = 10\,mV$$

$$v_{out\,cm} = A_c \cdot v_{cm} = \frac{A_d}{CMRR_{eff}} \cdot v_{cm}$$

$$= \frac{1}{50} \cdot 10\,mV = 200\,\mu V$$

(c) [2 pts] Find the value for R_o to limit the short-circuit current on the DRL electrode to no more than $10\,\mu A$ for a ± 3.3 V double rail voltage supply.

$$R_o = \frac{|\pm 3.3V|}{10\,\mu A} = 330\,k\Omega$$

10. **[4 pts] Guest lectures**

(a) *Real-world EEG brain-computer interfaces (BCI)*

i. [1 pt] List at least one of the factors that have limited practical commercial use of BCI.

The achievable bit rates are excruciatingly slow!
(one ASCII character every several seconds)

ii. [1 pt] What signal do you observe in the EEG on the occipital lobe when presenting a 15 Hz periodic flashing stimulus in the visual field of the subject?

A 15 Hz periodic EEG signal
(steady-state visually evoked potential, or "SSVEP")

(b) *Global health and wearables*

i. [1 pt] Explain what is meant by "reverse tropicalization."

If the design is rugged enough to function in the tropics, it will surely function in western society.

ii. [1 pt] What is the least number of electrodes that a wearable sensor needs to have in order to be able to monitor the full 6-lead frontal ECG?

Three (e.g. RA, LA, LL)