

# BENG 186B Winter 2020 Final

Thursday, March 19, 2020

*Last Name, First Name:* \_\_\_\_\_

- This final is open notes, including the class on-line materials. You may also use a calculator for algebra and arithmetic.
- This final has 21 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.
- There are 10 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- Circle or box your final answers and show your work on the pages provided.

1	/16
2	/12
3	/18
4	/6
5	/6
6	/8
7	/12
8	/8
9	/10
10	/4
<b>Total</b>	<b>/100</b>

You may find the following equations useful:

$$\omega_n = \frac{1}{\sqrt{LC}} \quad \zeta = \frac{1}{2}RC\omega_n$$

$$R = R_G(1 + G\epsilon) \quad \sigma = E\epsilon$$

$$V_o = A_d V_d + A_c V_{cm}$$

$$V_d = V_b - V_a$$

$$V_{cm} = (V_b + V_a)/2$$

$$V = \mathbf{M} \cdot \mathbf{r} = \cos\theta |\mathbf{M}| |\mathbf{r}|$$

$$\mathbf{e} = \int_0^\ell \mathbf{v} \times \mathbf{B} d\ell$$

$$\Delta f = \frac{v}{c}(\cos\theta_r + \cos\theta_s)f_s$$

$$V = E_{\text{glass}} - E_{\text{ref}} + E_{\text{Nernst}}$$

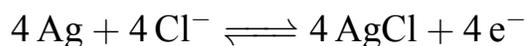
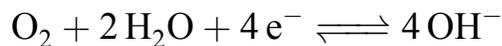
$$E_{\text{Nernst}} = \frac{RT}{nF} \ln(10) \log_{10} \left( \frac{[A^n]_{\text{out}}}{[A^n]_{\text{in}}} \right) \text{ for some ion } A^n \text{ with valence } n$$

$$V_m = \frac{RT}{F} \ln(10) \log_{10} \frac{P_{\text{Na}}[\text{Na}^+]_o + P_{\text{K}}[\text{K}^+]_o + P_{\text{Cl}}[\text{Cl}^-]_i}{P_{\text{Na}}[\text{Na}^+]_i + P_{\text{K}}[\text{K}^+]_i + P_{\text{Cl}}[\text{Cl}^-]_o}$$

$$\frac{RT}{F} \ln(10) = 62 \text{ mV at room temperature}$$

$$I = 4F [\text{O}_2] \phi \quad F = 96485 \text{ C/mol}$$

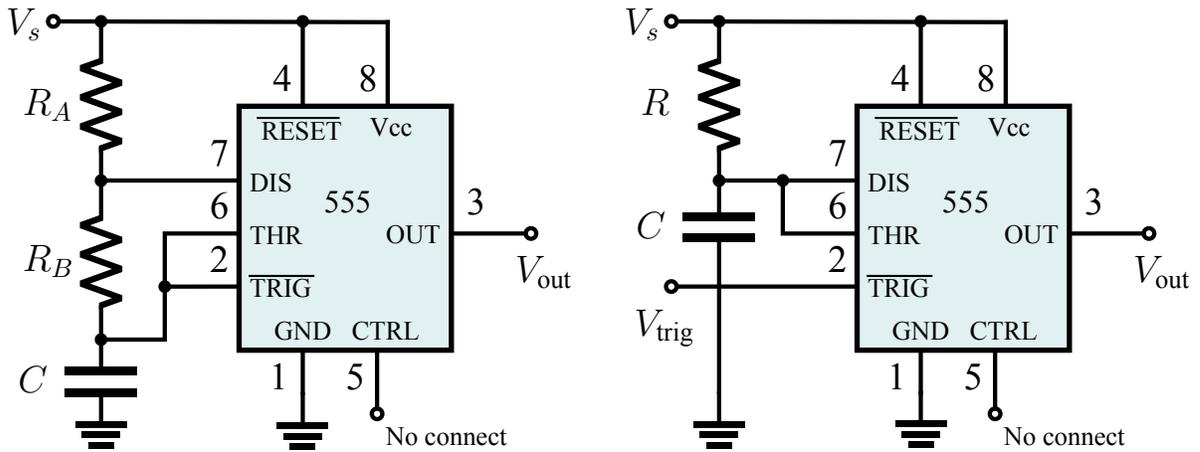
$$\log_{10} \text{PCO}_2 = -\text{pH} + \text{constant}$$



$$A(\lambda) = W L a(\lambda)$$

$$I_d = \frac{1}{1 - \exp(-d/\tau)} I_r$$

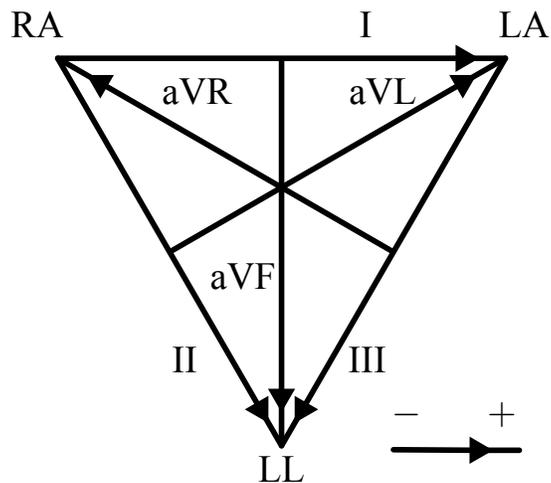
The following diagrams may come handy as well:



$$T_{hi} = 0.7(R_A + R_B)C$$

$$T_{lo} = 0.7R_B C$$

$$T = 1.1RC$$



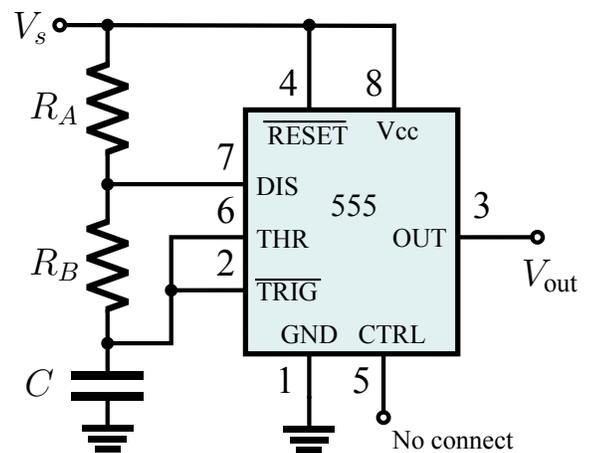
1. [16 pts] Circle the **best answer (only one answer per question)**:

(a) [1 pt] A digital temperature sensor with an accuracy of 1 % over the full range requires an analog-to-digital converter with at least:

- i. 6-bit resolution
- ii. 8-bit resolution
- iii. 8-bit precision
- iv. 10-bit sensitivity
- v. none of the above

(b) [1 pt] The circuit shown below implements what function?

- i. Rectifier
- ii. Astable multivibrator
- iii. Monostable single-shot
- iv. Hysteretic comparator
- v. None of the above



(c) [1 pt] Action potentials are initiated by:

- i. Slow activation of potassium
- ii. Slow inactivation of sodium
- iii. Fast activation of sodium
- iv. Fast inactivation of potassium
- v. None of the above

(d) [1 pt] The transient response of the retina to illumination is derived from electrophysiological measurement of the:

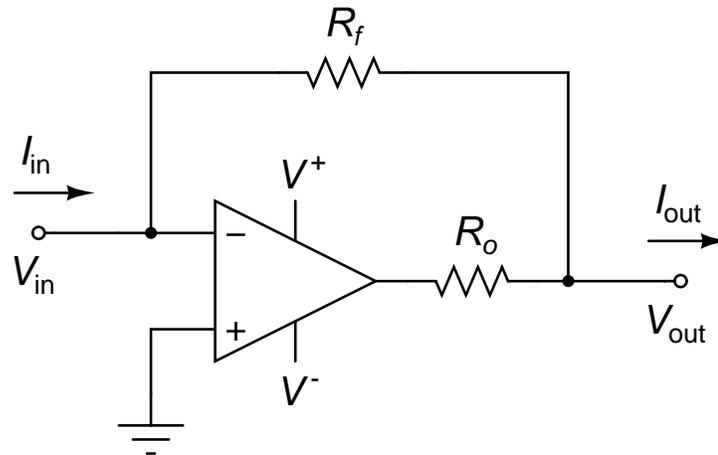
- i. ENG
- ii. EMG
- iii. ERG
- iv. EOG
- v. None of the above

- (e) [1 pt] The saturation of carbon dioxide in the blood can be derived from measurement of:
- i.  $PO_2$  and temperature
  - ii.  $PO_2$  and  $PCO_2$
  - iii.  $PCO_2$  and pH and temperature
  - iv.  $SO_2$
  - v. All of the above.
- (f) [1 pt] The Clark electrode:
- i. Measures saturation of oxygen in the blood.
  - ii. Measures the vector electrocardiogram.
  - iii. Measures acidity caused by carbon dioxide content.
  - iv. Produces hydrogen peroxide in amperometric sensing of oxygen flow.
  - v. None of the above.
- (g) [1 pt] Which one of these devices allows fast and accurate measurement of the blood pressure waveform?
- i. A micro-tipped manometer.
  - ii. A fluid-filled catheter.
  - iii. A sphygmomanometer.
  - iv. A tonometer.
  - v. All of the above.
- (h) [1 pt] Macroshock:
- i. Is more severe than microshock.
  - ii. Occurs when current entering the body is highly localized, with major isolated effect on mostly one organ.
  - iii. Occurs when current entering the body spreads across the body, reaching across multiple organs.
  - iv. Is the major cause of cardiac arrest.
  - v. None of the above.

(i) [8 pts] Indicate for each statement below whether it is true or false:

- i. **TRUE / FALSE:** A thermistor produces a voltage directly proportional to temperature.
- ii. **TRUE / FALSE:** An ideal transformer transfers electrical power from the primary to the secondary coil without loss in energy.
- iii. **TRUE / FALSE:** The current dipole generates an electric potential proportional to current, inversely proportional to distance squared, and proportional to the direction cosine between the dipole and the electrode.
- iv. **TRUE / FALSE:** The Ag/AgCl electrode is a non-polarizable electrode.
- v. **TRUE / FALSE:** The P wave in the electrocardiogram manifests ventricular repolarization.
- vi. **TRUE / FALSE:** An overdamped second-order system settles fast to input transients.
- vii. **TRUE / FALSE:** The rheobase current is the maximum current that allows the subject to withdraw.
- viii. **TRUE / FALSE:** Body size and weight are important to physiological effects.

2. [12 pts] Analyze the current-in, voltage-out circuit shown below. You may assume the opamp is ideal.

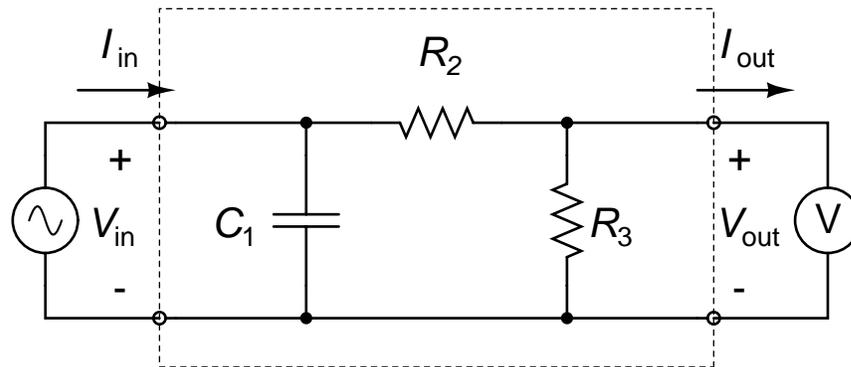


- (a) [1 pt] Find the input impedance.
- (b) [1 pt] Find the output impedance.
- (c) [4 pts] Find the output voltage  $V_{out}$  as a function of the input current  $I_{in}$ , and the corresponding transimpedance gain of the circuit.

(d) [3 pt] Find the range of input current  $I_{in}$  for which the output  $V_{out}$  does not saturate.

(e) [3 pt] You accidentally short the output  $V_{out}$  to the  $V^+$  supply. Find the short-circuit current  $I_{out}$ , assuming zero input current,  $I_{in} = 0$ .

3. [18 pts] Now analyze the following voltage-in, voltage-out filter circuit, with zero-error tolerance passive components:



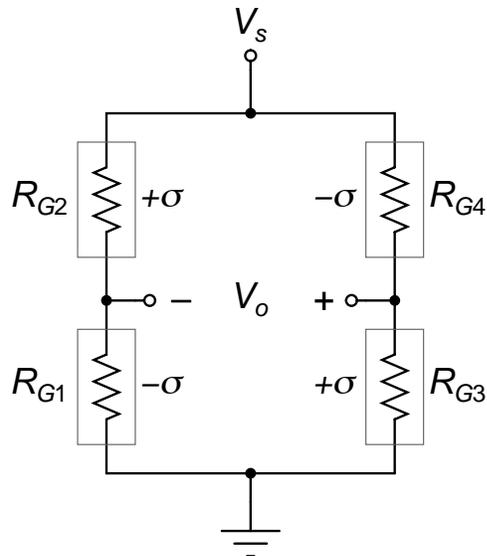
- (a) [1 pts] What is the impedance of the ideal voltage source driving the input of the filter?
- (b) [1 pts] What is the impedance of the ideal voltage meter loading the output of the filter?
- (c) [4 pt] Find the transfer function  $H(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$ . Does it depend on frequency?

(d) [3 pts] Find the input impedance  $Z_{in}(j\omega)$  of the filter.

(e) [3 pts] Find the output impedance  $Z_{out}(j\omega)$  of the filter.

- (f) [6 pt] Sketch the Bode plot of the input impedance  $Z_{in}(j\omega)$  from 0.1 Hz to 10 kHz, for  $C_1 = 16$  nF and  $R_2 = R_3 = 50$  k $\Omega$ . Be sure to indicate all units.

4. [6 pts] A Wheatstone bridge with two complementary pairs of strain gauges as shown below is used to transduce a stress  $\sigma$  to an output voltage  $V_o$ . All four strain gauges are identical with Young's modulus  $E$ , gauge factor  $G$ , nominal resistance  $R_G$ , and temperature coefficient  $\alpha$ .



- (a) [4 pts] Find the output voltage  $V_o$  as a function of stress  $\sigma$ , and the corresponding sensitivity of the output voltage to stress.

- (b) [2 pt] Now find the sensitivity of the output voltage to temperature.

5. [6 pts] You are given an ECG instrument that measures voltages  $RA$ ,  $LA$ , and  $LF$  on the right arm, left arm, and left foot respectively. It also measures voltages  $V1$  through  $V6$  horizontally along the chest, as per clinical-grade ECG guidelines for the 6 leads in the transversal plane.

(a) [4 pts] Find the following 4 leads of the frontal ECG in terms of the measured voltages:

$$I =$$

$$II =$$

$$aVL =$$

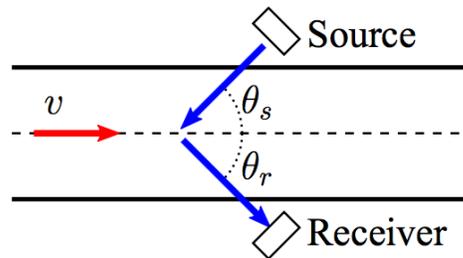
$$aVF =$$

(b) [2 pt] You find that the  $V3$  lead of the instrument is broken. What is your best guesstimate of  $V3$  in terms of the measured  $V2$  and  $V4$ ?

6. **[8 pts]** Design a simple ECG R-wave heartbeat detector that rejects low-frequency components in the ECG voltage signal by 6 dB per octave below 0.1 Hz cut-off frequency, amplifies the residual signal with a gain of 100, compares the amplified signal with  $\pm 10$  mV hysteresis around zero, and produces a 100 ms pulse of 3 V amplitude for every positive crossing in the comparison. You may use any active devices that you learned in class, any combination of resistors and capacitors, and a single 3 V battery. Specify values for all components.

7. [12 pts] A wide variety of non-invasive bioinstruments are available to monitor physiological signals as indicators of cardiovascular health. Here we consider two of these: a Doppler effect blood flow meter, and a pulse oximeter.

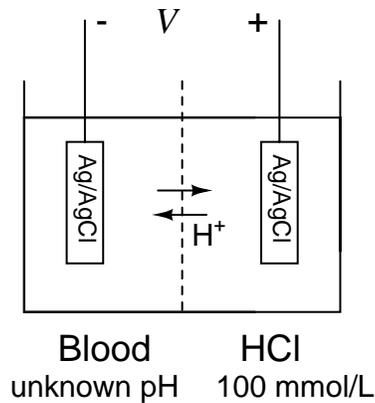
(a) [5 pts] Find the sensitivity of the Doppler shift  $\Delta f$  (in Hz) to volumetric blood flow  $\Phi$  (in L/s) for a Doppler ultrasonic transducer with source at frequency  $f = 3$  MHz, oriented at diagonal transmit and receiver angles to the blood vessel as shown ( $\theta_s = \pi/4$ ;  $\theta_r = -\pi/4$ ). Ignoring any viscosity, assume the blood flows uniformly across the vessel with radius  $R = 0.5$  cm. Ultrasonic waves propagate in blood and tissue at velocity  $c = 1,500$  m/s.



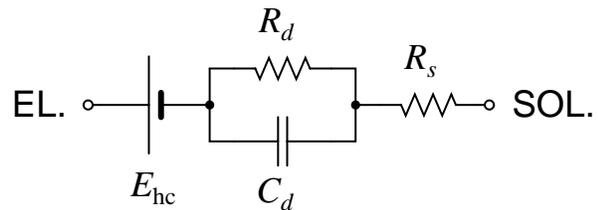
(b) [3 pts] Now consider the effect of blood viscosity in the vessel. You observe a continuous range of Doppler shifts from zero to a peak value  $\Delta f_{peak}$ . Explain.

(c) [4 pts] A pulse oximeter is widely used for non-invasive measurement of oxygen saturation in the blood. A similar instrument can be used to measure carbon dioxide saturation. Show how to obtain  $SCO_2$  from measurement of total absorbances  $A(\lambda_1)$  and  $A(\lambda_2)$  at two optical wavelengths  $\lambda_1$  and  $\lambda_2$ , and from knowledge of specific absorptivities  $a_c(\lambda)$  and  $a_r(\lambda)$  for carbonated and reduced hemoglobin, respectively, as a function of wavelength. You may assume the measurement is performed in the veins rather than the arteries, so blood oxygenation is negligible.

8. [8 pts] A pH meter, shown below, is used to measure acidity level in a sample of blood. One Ag/AgCl electrode is immersed in one chamber with the blood sample at room temperature, and a second Ag/AgCl electrode inside a separate chamber containing a solution of 100 mmol/L HCl in water. The glass membrane separating the two chambers is permeable to  $H^+$  only. For each Ag/AgCl electrode, the half cell potential is  $E_{hc} = 0.223\text{ V}$  and the impedance parameters are  $R_d = 10\text{ k}\Omega$ ,  $C_d = 1\text{ nF}$ , and  $R_s = 100\ \Omega$ .



Each Ag/AgCl electrode:

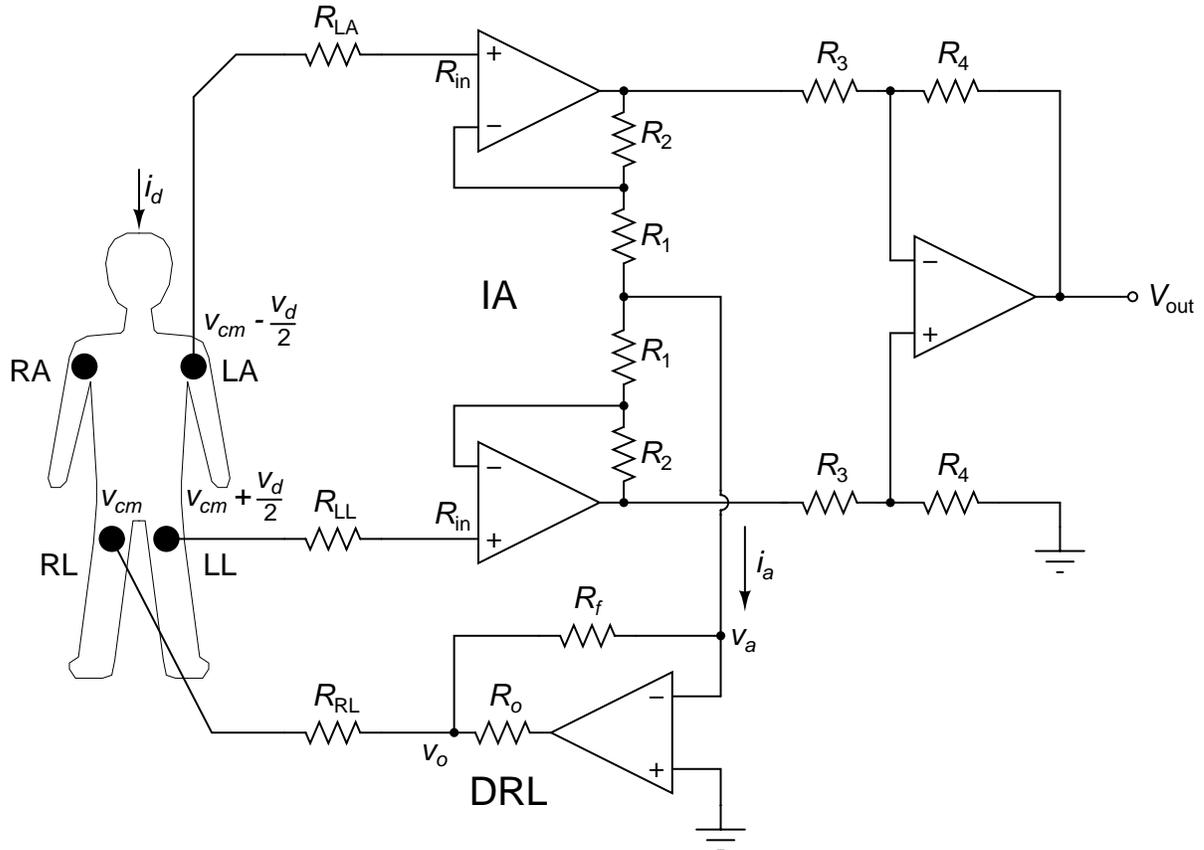


- (a) [2 pts] Find the pH inside the second chamber containing HCl.
- (b) [2 pts] The voltage  $V$  on the electrode in the HCl chamber relative to the electrode in the blood chamber measures  $-400\text{ mV}$ . Find the pH of the blood sample.

(c) [2 pts] If the Ag/AgCl electrode in the HCl chamber were replaced with a saturated calomel electrode, how would this affect the measurement of pH? Explain briefly.

(d) [2 pt] How are pH and  $PCO_2$  related, and what other factors are important? Explain briefly.

9. [10 pts] Consider the combined instrumentation amplifier (IA) and driven right leg (DRL) system below with resistances  $R_1 = 1\text{ k}\Omega$ ,  $R_2 = 99\text{ k}\Omega$ ,  $R_3 = 10\text{ k}\Omega$ ,  $R_4 = 100\text{ k}\Omega$ , and  $R_f = 500\text{ k}\Omega$ , and with electrode resistances  $R_{LA} = 95\text{ k}\Omega$ ,  $R_{LL} = 105\text{ k}\Omega$ , and  $R_{RL} = 100\text{ k}\Omega$ . All opamp input impedances are  $R_{in} = 10\text{ G}\Omega$ .



- (a) [3 pts] Find the voltage  $v_a$  at the DRL input node, and the current  $i_a$  entering the DRL input node in terms of the body common-mode voltage  $v_{cm}$ .

(b) [3 pts] Find the corresponding effective right-leg resistance  $R_{RL\text{ eff}} = v_{cm}/i_d$ .

(c) [2 pts] Find the differential gain  $A_d$ , and effective common-mode rejection ratio  $\text{CMRR}_{\text{eff}}$ .

(d) [2 pts] For a displacement current with amplitude  $i_d = 10 \mu\text{A}$  entering the body, and an ECG signal with amplitude  $v_d = 100 \mu\text{V}$ , find the signal-to-noise ratio at the output of the instrumentation amplifier.

