BENG 186B Winter 2020 Final

Thursday, March 19, 2020

Last Name, First Name: SOLUTIONS

- This final is open notes, including the class on-line materials. You may also use a calculator for algebra and arithmetic.
- This final has 21 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.
- There are 10 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- Circle or box your final answers and show your work on the pages provided.

1	/16
2	/12
3	/18
4	/6
5	/6
6	/8
7	/12
8	/8
9	/10
10	/4
Total	**/100**

You may find the following equations useful:

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad \zeta = \frac{1}{2} RC\, \omega_n$$

$$R = R_G\,(1 + G\,\epsilon) \qquad \sigma = E\,\epsilon$$

$$V_o = A_d V_d + A_c V_{cm}$$

$$V_d = V_b - V_a$$

$$V_{cm} = (V_b + V_a)/2$$

$$V = \mathbf{M} \cdot \mathbf{r} = \cos\theta\,|\mathbf{M}|\,|\mathbf{r}|$$

$$e = \int_0^\ell \mathbf{v} \times \mathbf{B}\, d\ell$$

$$\Delta f = \frac{v}{c}(\cos\theta_r + \cos\theta_s) f_s$$

$$V = E_{\text{glass}} - E_{\text{ref}} + E_{\text{Nernst}}$$

$$E_{\text{Nernst}} = \frac{RT}{nF} \ln(10) \log_{10} \left(\frac{[\mathrm{A^n}]_{\text{out}}}{[\mathrm{A^n}]_{\text{in}}} \right) \text{ for some ion } \mathrm{A^n} \text{ with valence } n$$

$$V_m = \frac{RT}{F} \ln(10) \log_{10} \frac{P_{\text{Na}}[\mathrm{Na^+}]_o + P_{\text{K}}[\mathrm{K^+}]_o + P_{\text{Cl}}[\mathrm{Cl^-}]_i}{P_{\text{Na}}[\mathrm{Na^+}]_i + P_{\text{K}}[\mathrm{K^+}]_i + P_{\text{Cl}}[\mathrm{Cl^-}]_o}$$

$$\frac{RT}{F} \ln(10) = 62\,\text{mV at room temperature}$$

$$I = 4F\,[\mathrm{O_2}]\,\phi \qquad F = 96\,485\,\mathrm{C/mol}$$

$$\log_{10} \mathrm{PCO_2} = -\mathrm{pH} + \text{constant}$$

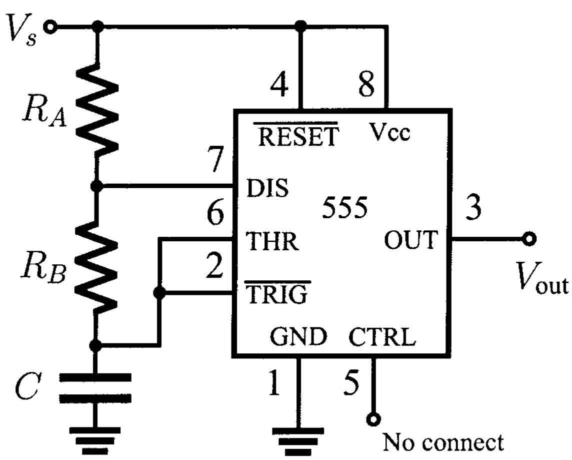
$$\mathrm{O_2 + 2\,H_2O + 4\,e^- \rightleftharpoons 4\,OH^-}$$

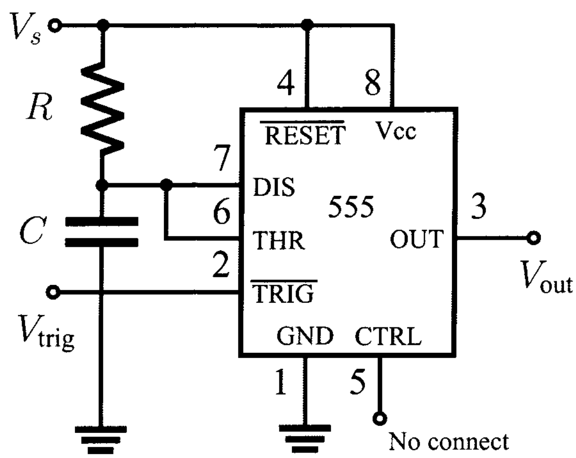
$$\mathrm{4\,Ag + 4\,Cl^- \rightleftharpoons 4\,AgCl + 4\,e^-}$$

$$A(\lambda) = W\,L\,a(\lambda)$$

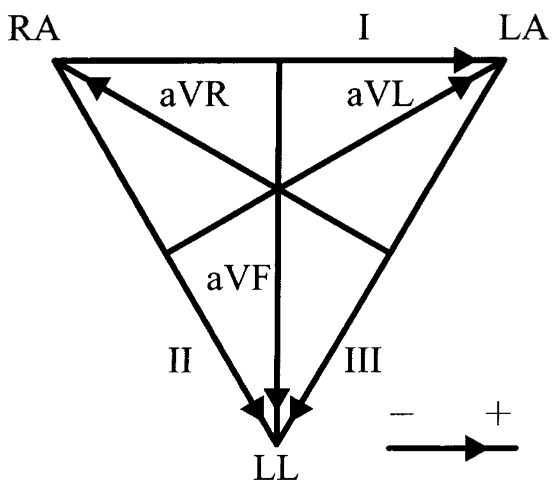
$$I_d = \frac{1}{1-\exp(-d/\tau)}\,I_r$$

The following diagrams may come handy as well:





$T_{\text{hi}} = 0.7(R_A + R_B)C$ $\qquad$ $T_{\text{lo}} = 0.7R_BC$ $\qquad$ $T = 1.1RC$



1. **[16 pts]** Circle the **best answer (only one answer per question)**:

(a) [1 pt] A digital temperature sensor with an accuracy of 1 % over the full range requires an analog-to-digital converter with at least:

i. 6-bit resolution

ii. 8-bit resolution

iii. 8-bit precision

iv. 10-bit sensitivity

v. none of the above

$\frac{1}{2}$ LSB $= \frac{1}{2} 2^{-6} = \frac{1}{128} < 1\%$ OK!

(b) [1 pt] The circuit shown below implements what function?

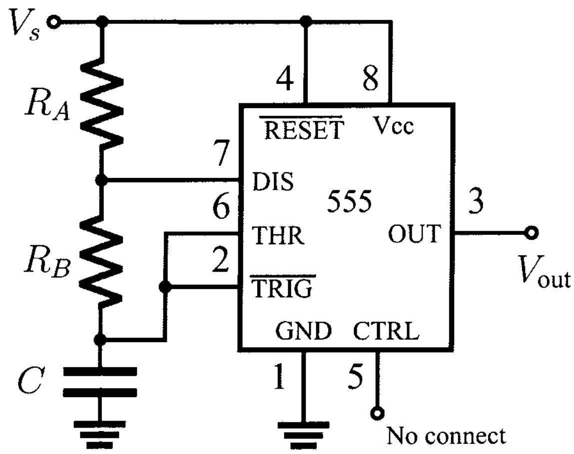
i. Rectifier

ii. Astable multivibrator

iii. Monostable single-shot

iv. Hysteretic comparator

v. None of the above



(c) [1 pt] Action potentials are initiated by:

i. Slow activation of potassium

ii. Slow inactivation of sodium

iii. Fast activation of sodium

iv. Fast inactivation of potassium

v. None of the above

(d) [1 pt] The transient response of the retina to illumination is derived from electrophysiological measurement of the:

i. ENG

ii. EMG

iii. ERG

iv. EOG

v. None of the above

(e) [1 pt] The saturation of carbon dioxide in the blood can be derived from measurement of:

i. PO_2 and temperature
ii. PO_2 and PCO_2
iii. PCO_2 and pH and temperature
iv. SO_2
v. All of the above.

(f) [1 pt] The Clark electrode:

i. Measures saturation of oxygen in the blood. (no, partial pressure)
ii. Measures the vector electrocardiogram.
iii. Measures acidity caused by carbon dioxide content.
iv. Produces hydrogen peroxide in amperometric sensing of oxygen flow.
v. None of the above.

(g) [1 pt] Which one of these devices allows fast and accurate measurement of the blood pressure waveform?

i. A micro–tipped manometer.
ii. A fluid–filled catheter.
iii. A sphygmomanometer.
iv. A tonometer.
v. All of the above.

(h) [1 pt] Macroshock:

i. Is more severe than microshock.
ii. Occurs when current entering the body is highly localized, with major isolated effect on mostly one organ.
iii. Occurs when current entering the body spreads across the body, reaching across multiple organs.
iv. Is the major cause of cardiac arrest.
v. None of the above.

(i) [8 pts] Indicate for each statement below whether it is true or false:

i. **TRUE** **FALSE**: A thermistor produces a voltage directly proportional to temperature. (resistance)

ii. **TRUE** **FALSE**: An ideal transformer transfers electrical power from the primary to the secondary coil without loss in energy.

iii. **TRUE** **FALSE**: The current dipole generates an electric potential proportional to current, inversely proportional to distance squared, and proportional to the direction cosine between the dipole and the electrode.

iv. **TRUE** **FALSE**: The Ag/AgCl electrode is a non-polarizable electrode.

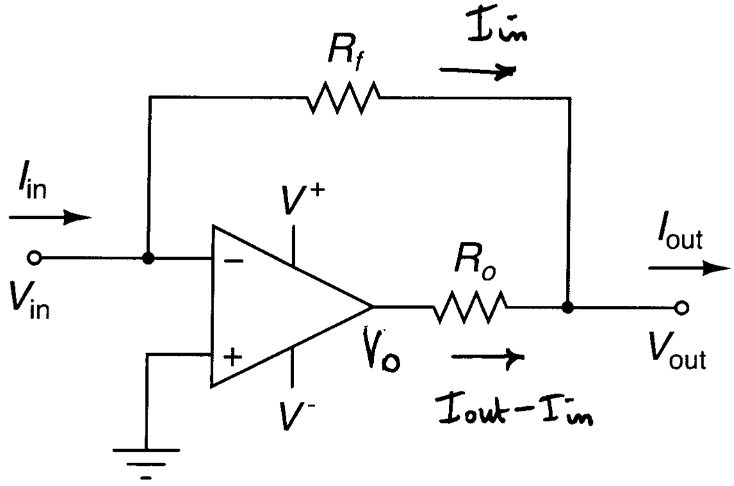
v. **TRUE** **FALSE**: The P wave in the electrocardiogram manifests ventrical repolarization. (atrial depolarization)

vi. **TRUE** **FALSE**: An overdamped second-order system settles fast to input transients. (slow)

vii. **TRUE** **FALSE**: The rheobase current is the maximum current that allows the subject to withdraw. (let-go)

viii. **TRUE** **FALSE**: Body size and weight are important to physiological effects.

2. **[12 pts]** Analyze the current-in, voltage-out circuit shown below. You may assume the opamp is ideal.



(a) [1 pt] Find the input impedance.

0 ($V_{in} = 0$ regardless of I_{in})

(b) [1 pt] Find the output impedance.

0 (V_{out} is independent of I_{out})

(c) [4 pts] Find the output voltage V_{out} as a function of the input current I_{in}, and the corresponding transimpedance gain of the circuit.

KCL @ V_{in} node: $I_{in} = \frac{0 - V_{out}}{R_f}$

$\Rightarrow V_{out} = -R_f \cdot I_{in}$

Transimpedance gain: $\frac{dV_{out}}{dI_{in}} = -R_f$

(d) [3 pt] Find the range of input current I_{in} for which the output V_{out} does not saturate.

For an unloaded output ($I_{out} = 0$), the opamp output $V_0 = -(R_f + R_o) I_{in}$ does not saturate (reach V^- or V^+) provided

$$-\frac{V^+}{R_f+R_o} \leq I_{in} \leq -\frac{V^-}{R_f+R_o}$$

(see diagram on the previous page)

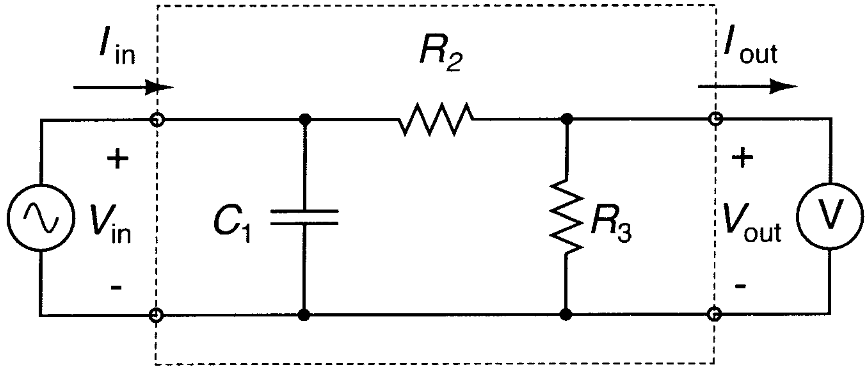
(e) [3 pt] You accidentally short the output V_{out} to the V^+ supply. Find the short-circuit current I_{out}, assuming zero input current, $I_{in} = 0$.

For zero input current, the opamp inverting input is at the same voltage as the shorted output, $V_{out} = V^+$.

$\Rightarrow V_o = V^-$, and

$$I_{out} = \frac{V_o - V_{out}}{R_o} = -\frac{V^+ - V^-}{R_o}$$

3. **[18 pts]** Now analyze the following voltage-in, voltage-out filter circuit, with zero-error tolerance passive components:



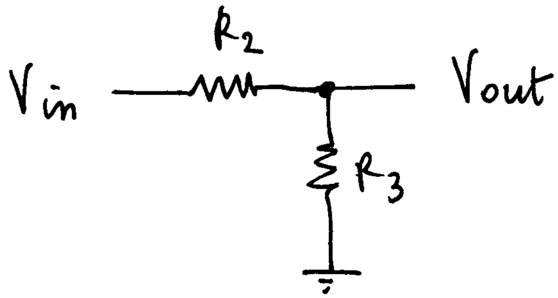
(a) [1 pts] What is the impedance of the ideal voltage source driving the input of the filter?

0

(b) [1 pts] What is the impedance of the ideal voltage meter loading the output of the filter?

∞

(c) [4 pt] Find the transfer function $H(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$. Does it depend on frequency?

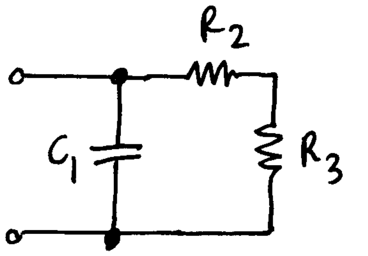


(C_1 is driven directly by the source V_{in}, and does not load the output.)

Standard voltage divider:

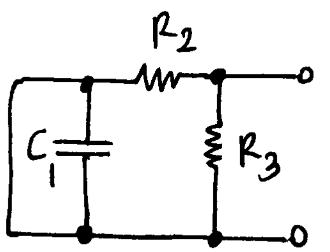
$$H = \frac{V_{out}}{V_{in}} = \frac{R_3}{R_2 + R_3}$$ independent of frequency

(d) [3 pts] Find the input impedance $Z_{in}(j\omega)$ of the filter.



$$Z_{in} = \frac{1}{j\omega C_1} \parallel (R_2 + R_3) = \frac{R_2 + R_3}{1 + j\omega (R_2 + R_3) C_1}$$

(e) [3 pts] Find the output impedance $Z_{out}(j\omega)$ of the filter.



$$Z_{out} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

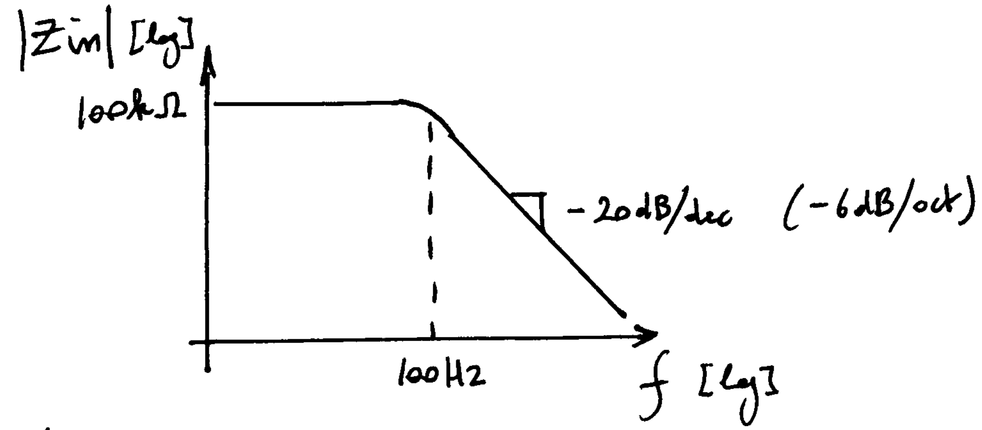
(f) [6 pt] Sketch the Bode plot of the input impedance $Z_{in}(j\omega)$ from 0.1 Hz to 10 kHz, for $C_1 = 16$ nF and $R_2 = R_3 = 50$ kΩ. Be sure to indicate all units.

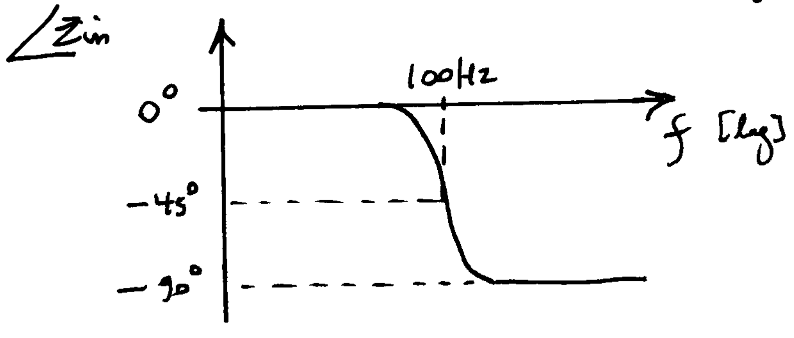
$$Z_{in}(j\omega) = \frac{R_2 + R_3}{1 + j\omega(R_2+R_3)C_1}$$

First-order lowpass with:

DC gain: $Z_{in}(0) = R_2 + R_3$

Cutt-off frequency: $f_c = \frac{1}{2\pi(R_2+R_3)C_1} = \frac{1}{6.28 \; 100k\Omega \; 16nF} = 100\,\text{Hz}$

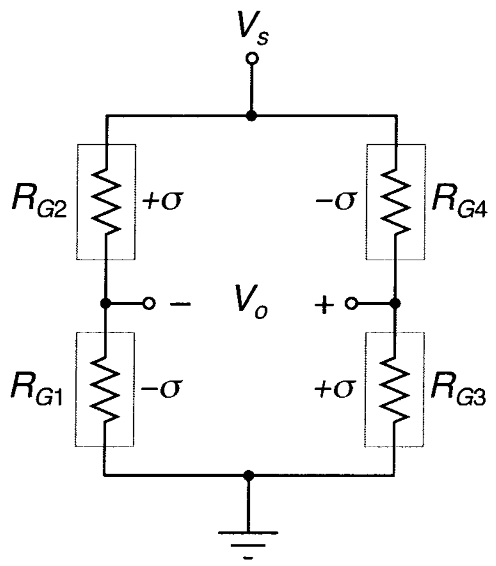




4. **[6 pts]** A Wheatstone bridge with two complementary pairs of strain gauges as shown below is used to transduce a stress σ to an output voltage V_o. All four strain gauges are identical with Young's modulus E, gauge factor G, nominal resistance R_G, and temperature coefficient α.

$R_G(1+\frac{G}{E}\sigma) = R_{G2}$ $\qquad$ $R_{G4} = R_G(1-\frac{G}{E}\sigma)$

$R_G(1-\frac{G}{E}\sigma) = R_{G1}$ $\qquad$ $R_{G3} = R_G(1+\frac{G}{E}\sigma)$



(a) [4 pts] Find the output voltage V_o as a function of stress σ, and the corresponding sensitivity of the output voltage to stress.

$$V_o = \left(\frac{R_{G3}}{R_{G3}+R_{G4}} - \frac{R_{G1}}{R_{G1}+R_{G2}}\right)V_s = \left(\frac{1+\frac{G}{E}\sigma}{2} - \frac{1-\frac{G}{E}\sigma}{2}\right)V_s = \frac{G}{E}V_s\sigma$$

$$S = \frac{dV_o}{d\sigma} = \frac{G}{E}V_s$$

(b) [2 pt] Now find the sensitivity of the output voltage to temperature.

0 , since all gauges have identical temperature coefficient α.

5. **[6 pts]** You are given an ECG instrument that measures voltages RA, LA, and LF on the right arm, left arm, and left foot respectively. It also measures voltages $V1$ through $V6$ horizontally along the chest, as per clinical-grade ECG guidelines for the 6 leads in the transversal plane.

(a) [4 pts] Find the following 4 leads of the frontal ECG in terms of the measured voltages:

(p. 3)

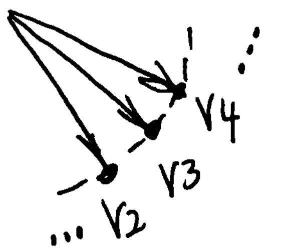
$$\mathrm{I} = LA - RA$$

$$\mathrm{II} = LL - RA$$

$$\mathrm{aVL} = LA - \frac{1}{2}(RA + LL)$$

$$\mathrm{aVF} = LL - \frac{1}{2}(RA + LA)$$

(b) [2 pt] You find that the $V3$ lead of the instrument is broken. What is your best guesstimate of $V3$ in terms of the measured $V2$ and $V4$?

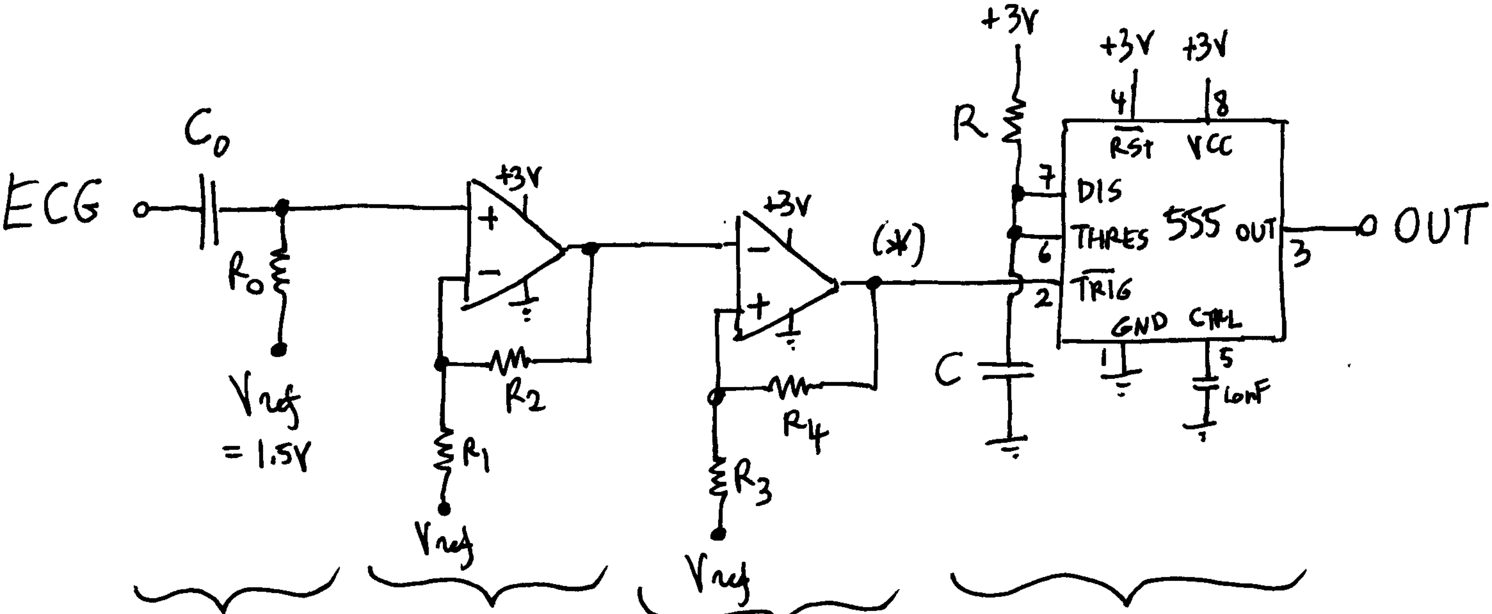


Linear interpolation:

$V3 \approx$ midway between $V2$ and $V4$

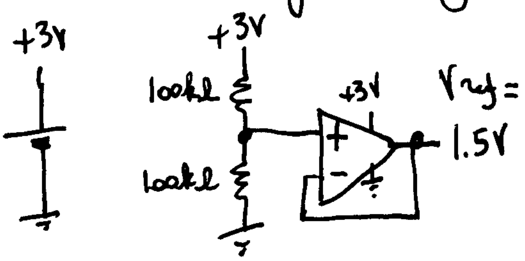
$\approx \frac{1}{2}(V_2 + V_4)$

6. **[8 pts]** Design a simple ECG R-wave heartbeat detector that rejects low-frequency components in the ECG voltage signal by 6 dB per octave below 0.1 Hz cut-off frequency, amplifies the residual signal with a gain of 100, compares the amplified signal with ±10 mV hysteresis around zero, and produces a 100 ms pulse of 3 V (*) amplitude for every positive crossing in the comparison. You may use any active devices that you learned in class, any combination of resistors and capacitors, and a single 3 V battery. Specify values for all components.



HIGHPASS	NON-INVERTING AMPLIFIER	INVERTING HYSTERETIC COMPARATOR	MONOSTABLE
$\frac{1}{2\pi R_0 C_0} = 0.1\ Hz$	$1 + \frac{R_2}{R_1} = 100$	$\pm \frac{R_3}{R_3 + R_4} V_{ref} = \pm 10mV$	$1.1\ RC = 100ms$
e.g.: $R_0 = 1M\Omega$, $C_0 = 1.6\mu F$	$R_1 = 1k\Omega$, $R_2 = 99k\Omega$	$R_3 = 1k\Omega$, $R_4 = 149k\Omega$	$R = 90k\Omega$, $C = 1\mu F$

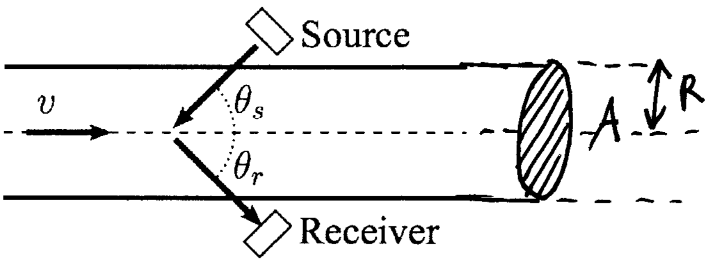
The signal reference V_{ref} is midway between ground and +3V to maximize signal range with symmetric positive & negative swing.



(*) A positive transition in the ECG over the +10mV threshold causes a low transition at the comparator output, which triggers the 555 monostable.

7. **[12 pts]** A wide variety of non-invasive bioinstruments are available to monitor physiological signals as indicators of cardiovascular health. Here we consider two of these: a Doppler effect blood flow meter, and a pulse oximeter.

(a) [5 pts] Find the sensitivity of the Doppler shift Δf (in Hz) to volumetric blood flow Φ (in L/s) for a Doppler ultrasonic transducer with source at frequency $f = 3$ MHz, oriented at diagonal transmit and receiver angles to the blood vessel as shown ($\theta_s = \pi/4$; $\theta_r = -\pi/4$). Ignoring any viscosity, assume the blood flows uniformly across the vessel with radius $R = 0.5$ cm. Ultrasonic waves propagate in blood and tissue at velocity $c = 1{,}500$ m/s.



$$\Phi = v \cdot A = v \cdot \pi R^2$$

and $$\frac{\Delta f}{f} = (\cos\theta_s + \cos\theta_r)\frac{v}{c} = \sqrt{2}\,\frac{v}{c}$$

$$\Rightarrow \Phi = \pi R^2 \frac{c}{\sqrt{2}} \frac{\Delta f}{f} \quad \text{or} \quad \Delta f = \frac{\sqrt{2} f}{\pi R^2 c} \cdot \Phi$$

$$S = \frac{d\Delta f}{d\Phi} = \frac{\sqrt{2} f}{\pi R^2 c} = \frac{1.41 \; 3\,\text{MHz}}{3.14 \; 25\, 10^{-6}\,\text{m}^2 \; 1500\,\frac{\text{m}}{\text{s}}}$$

$$= 36 \frac{\text{MHz}}{\text{m}^3/\text{s}} = 3.6\, 10^4 \frac{\text{Hz}}{\text{L/s}}$$

(b) [3 pts] Now consider the effect of blood viscosity in the vessel. You observe a continuous range of Doppler shifts from zero to a peak value Δf_{peak}. Explain.

For laminar flow, velocity ranges from 0 to v_{peak} across the vessel.

(Specifically, $v(\rho) = v_{peak} \cdot \left(1 - \frac{\rho^2}{R^2}\right)$, POISEUILLE)

(c) [4 pts] A pulse oximeter is widely used for non-invasive measurement of oxygen saturation in the blood. A similar instrument can be used to measure carbon dioxide saturation. Show how to obtain SCO_2 from measurement of total absorbances $A(\lambda_1)$ and $A(\lambda_2)$ at two optical wavelengths λ_1 and λ_2, and from knowledge of specific absorptivities $a_c(\lambda)$ and $a_r(\lambda)$ for carbonated and reduced hemoglobin, respectively, as a function of wavelength. You may assume the measurement is performed in the veins rather than the arteries, so blood oxygenation is negligible.

Same principle as SO_2:

$$A(\lambda_1) = WL\left(SCO_2\, a_c(\lambda_1) + (1 - SCO_2)\, a_r(\lambda_1)\right)$$

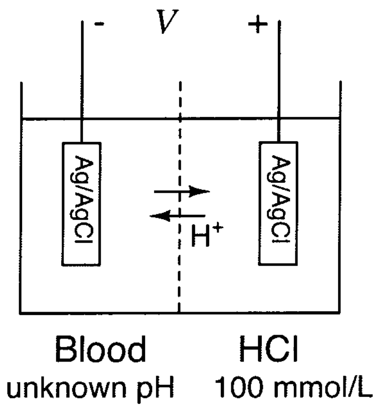
$$A(\lambda_2) = WL\left(SCO_2\, a_c(\lambda_2) + (1 - SCO_2)\, a_r(\lambda_2)\right)$$

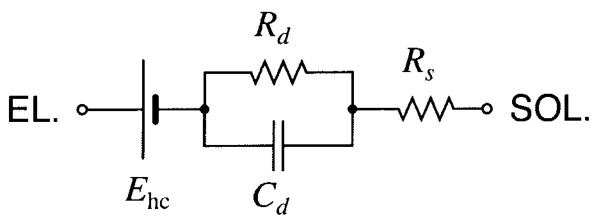
Choose λ_2 such that $a_c(\lambda_2) = a_r(\lambda_2) = a_i$ (ISOSBESTIC)

$$\Rightarrow \frac{A(\lambda_1)}{A(\lambda_2)} = \frac{a_r(\lambda_1)}{a_i} - \frac{a_r(\lambda_1) - a_c(\lambda_1)}{a_i}\, SCO_2$$

or $$SCO_2 = \frac{a_r(\lambda_1)}{a_r(\lambda_1) - a_c(\lambda_1)} - \frac{a_i}{a_r(\lambda_1) - a_c(\lambda_1)} \cdot \frac{A(\lambda_1)}{A(\lambda_2)}$$

8. **[8 pts]** A pH meter, shown below, is used to measure acidity level in a sample of blood. One Ag/AgCl electrode is immersed in one chamber with the blood sample at room temperature, and a second Ag/AgCl electrode inside a separate chamber containing a solution of 100 mmol/L HCl in water. The glass membrane separating the two chambers is permeable to H^+ only. For each Ag/AgCl electrode, the half cell potential is $E_{hc} = 0.223\,\text{V}$ and the impedance parameters are $R_d = 10\,\text{k}\Omega$, $C_d = 1\,\text{nF}$, and $R_s = 100\,\Omega$.





(a) [2 pts] Find the pH inside the second chamber containing HCl.

$$pH_{HCl} = -\log_{10} \underset{(\frac{mol}{l})}{[H^+]} = -\log_{10}(0.1) = 1$$

(b) [2 pts] The voltage V on the electrode in the HCl chamber relative to the electrode in the blood chamber measures -400 mV. Find the pH of the blood sample.

$$V = 62\,\text{mV} \cdot \log_{10} \frac{[H^+]_{blood}}{[H^+]_{HCl}} = 62\,\text{mV} \cdot (pH_{HCl} - pH_{blood}) = -400\,\text{mV}$$

$$\Rightarrow pH_{blood} = pH_{HCl} + \frac{400\,\text{mV}}{62\,\text{mV}} = 7.45$$

(c) [2 pts] If the Ag/AgCl electrode in the HCl chamber were replaced with a saturated camomel electrode, how would this affect the measurement of pH? Explain briefly.

V would additionally measure the difference in half-cell potentials, $E_{hc\ camomel} - E_{hc\ Ag/AgCl}$

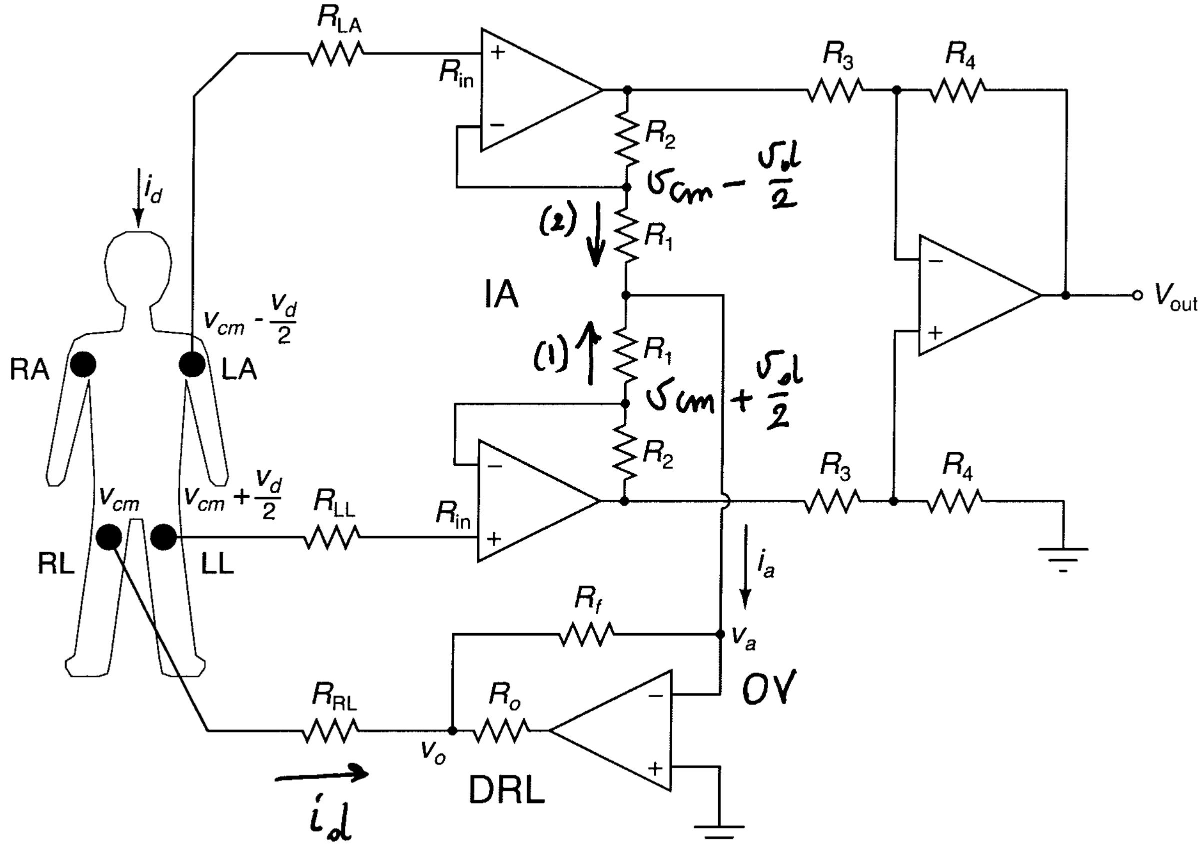
(d) [2 pt] How are pH and PCO_2 related, and what other factors are important? Explain briefly.

$[H^+]$ and $[CO_2]$ are linearly related, thus

$pH = -\log PCO_2 + cst$ that depends on $[HCO_3^-]$ and temperature

(Severinghaus)

9. **[10 pts]** Consider the combined instrumentation amplifier (IA) and driven right leg (DRL) system below with resistances $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 99\,\mathrm{k\Omega}$, $R_3 = 10\,\mathrm{k\Omega}$, $R_4 = 100\,\mathrm{k\Omega}$, and $R_f = 500\,\mathrm{k\Omega}$, and with electrode resistances $R_{LA} = 95\,\mathrm{k\Omega}$, $R_{LL} = 105\,\mathrm{k\Omega}$, and $R_{RL} = 100\,\mathrm{k\Omega}$. All opamp input impedances are $R_{in} = 10\,\mathrm{G\Omega}$.



(a) [3 pts] Find the voltage v_a at the DRL input node, and the current i_a entering the DRL input node in terms of the body common-mode voltage v_{cm}.

$$v_a = 0$$

$$i_a = \underbrace{\frac{v_{cm} + \frac{v_d}{2}}{R_1}}_{(1)} + \underbrace{\frac{v_{cm} - \frac{v_d}{2}}{R_1}}_{(2)} = \frac{2}{R_1} v_{cm}$$

(b) [3 pts] Find the corresponding effective right-leg resistance $R_{RL\,\mathrm{eff}} = v_{cm}/i_d$.

As usual: $v_o = -R_f\, i_d = -\dfrac{2R_f}{R_1} v_{cm}$

and $v_o = v_{cm} - R_{RL}\, i_d$

$$\Rightarrow R_{RL\,eff} = \frac{v_{cm}}{i_d} = \frac{R_{RL}}{1+\frac{2R_f}{R_1}} = \frac{100\,k\Omega}{1+\frac{1\,M\Omega}{1\,k\Omega}} = 100\,\Omega$$

(c) [2 pts] Find the differential gain A_d, and effective common-mode rejection ratio $\mathrm{CMRR_{eff}}$.

$$A_d = \left(1+\frac{R_2}{R_1}\right)\frac{R_4}{R_3} = (1+99)\,10 = 1{,}000$$

$$CMRR_{eff} = \frac{R_{in}}{|R_{LA}-R_{LL}|} = \frac{10\,G\Omega}{10\,k\Omega} = 10^6 \quad (120\,dB)$$

(d) [2 pts] For a displacement current with amplitude $i_d = 10\,\mu$A entering the body, and an ECG signal with amplitude $v_d = 100\,\mu$V, find the signal-to-noise ratio at the output of the instrumentation amplifier.

$$SNR_{out} = CMRR_{eff}\cdot\frac{v_d}{v_{cm}} = CMRR_{eff}\cdot\frac{v_d}{R_{RL\,eff}\cdot i_d}$$

$$= 10^6\,\frac{100\,\cancel{\mu V}}{100\,\Omega\cdot 10\,\cancel{\mu A}} = 10^5 \quad (100\,dB)$$

10. **[4 pts] Guest lectures**

(a) *Dry-electrode EEG systems*

i. [1 pt] List one example each of the advantages and disadvantages of dry electrodes over wet/gel-based electrodes for electroencephalography recording.

(+) Less mess, easy prep without the gel

(−) Worse motion artifacts

ii. [1 pt] List at least one example of signal artifacts that contaminate the EEG observed from electrodes mounted on the forehead.

Eye blinks, EOG, EMG, ...

(b) *Global health and wearables*

i. [1 pt] Explain what is meant by the double burden of disease in developing countries.

Having to cope with diseases both due to poor living conditions in the third world, and unhealthy life style with increasing wealth.

ii. [1 pt] Explain why a wristwatch is not able to measure any lead of the ECG, unless it is brought in contact with at least one more body part other than the limb on which it is worn.

There is no ECG potential difference within the limb itself.