

BENG 186B Winter 2013

Quiz 3

March 8, 2013

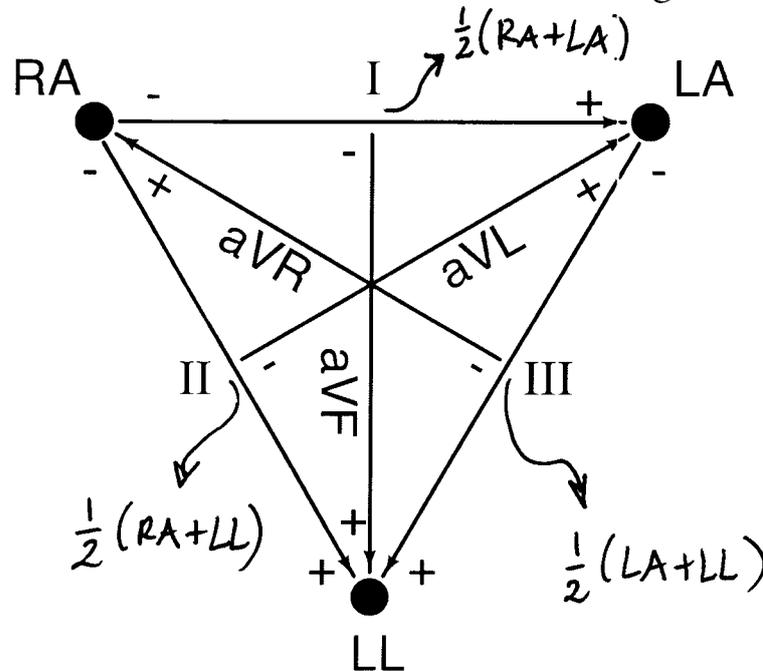
NAME (Last, First):

SOLUTIONS

- This quiz is closed book and closed note. You may use a calculator for algebra.
- Write your final answers in the space provided; show your work only on the pages provided.
- Do not attach separate sheets. If you need more space, use the back of the pages.
- Points for each problem are given in [brackets], 100 points total. The quiz is 50 minutes long.

1	/15
2	/40
3	/20
4	/25
Total	/100

1. [15 pts] Using Einthoven's Triangle, write the lead voltages I, II, III, aVR, aVL, and aVF in terms of the electrode voltages RA, LA, and LL.



$$I = \underline{LA - RA}$$

$$II = \underline{LL - RA}$$

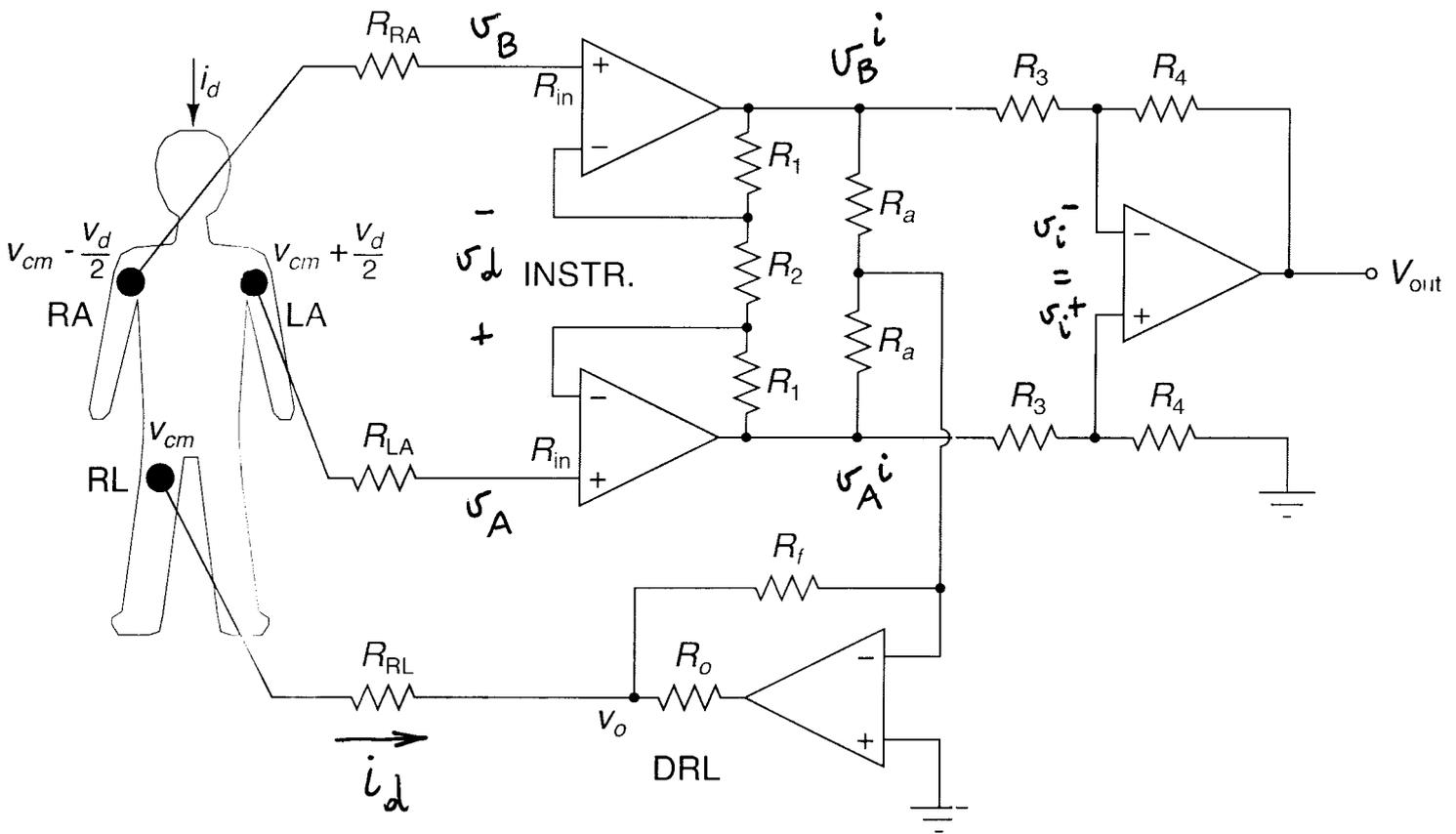
$$III = \underline{LL - LA}$$

$$aVR = \underline{RA - \frac{1}{2}(LA+LL)}$$

$$aVL = \underline{LA - \frac{1}{2}(RA+LL)}$$

$$aVF = \underline{LL - \frac{1}{2}(RA+LA)}$$

2. [40 pts] Consider a single-lead ECG instrumentation amplifier with driven right leg (DRL), and with three electrodes RA, LA and RL connected to the body as shown below. The electrode-skin interface resistances are $R_{RA} = 50 \text{ k}\Omega$, $R_{LA} = 150 \text{ k}\Omega$ and $R_{RL} = 100 \text{ k}\Omega$. The opamp input impedance is $R_{in} = 1 \text{ G}\Omega$ to ground. The instrumentation amplifier resistance values are $R_2 = R_3 = 1 \text{ k}\Omega$ and $R_1 = R_4 = 100 \text{ k}\Omega$, and the DRL resistance values are $R_a = 100 \text{ k}\Omega$, $R_f = 10 \text{ M}\Omega$, and $R_o = 1 \text{ M}\Omega$, all exactly (with zero tolerance).



a. [5 pts] What ECG lead does V_{out} represent? What is the polarity of the lead in V_{out} ?

Answer:

ECG Lead = I

Polarity = + (+/-)

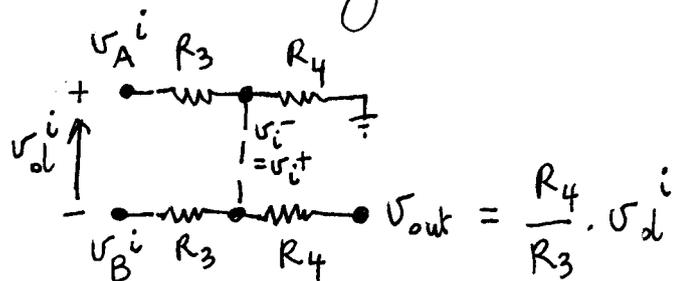
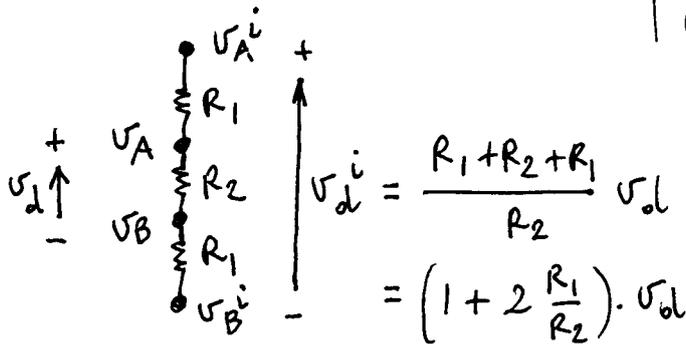
$$V_{out} = A_d \cdot v_d \quad \text{where } v_d = LA - RA = I \text{ and } A_d > 0$$

- b. [15 pts] Find the values for differential gain A_d and common-mode rejection ratio CMRR of the instrumentation amplifier connected to the RA and LA electrodes on the body.

Answer:

$$A_d = \underline{20,100} \quad (86 \text{ dB}) \quad \text{CMRR} = \underline{10,000} \quad (80 \text{ dB})$$

Standard instrumentation amplifier (non-inverting):



$$\Rightarrow V_{out} = \left(1 + 2 \frac{R_1}{R_2}\right) \frac{R_4}{R_3} \cdot V_d \quad \text{or} \quad A_d = \left(1 + 2 \frac{R_1}{R_2}\right) \frac{R_4}{R_3} = 20,100$$

$$A_c = 0 \quad (\text{just the amplifier})$$

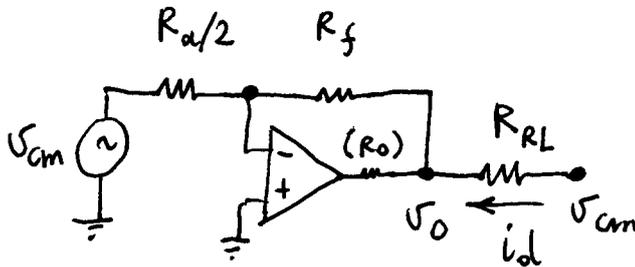
Effective CMRR, due to mismatch in electrode-skin impedances and finite input impedance of the amplifier:

$$\text{CMRR} \approx \frac{R_{in}}{|R_{LA} - R_{RA}|} = \frac{1 \text{ G}\Omega}{150 \text{ k}\Omega - 50 \text{ k}\Omega} = \frac{10^9}{10^5} = 10^4$$

- c. [10 pts] Find the driven right leg (DRL) effective electrode resistance, defined as $R_{RL\text{ eff}} = v_{cm} / i_d$.

Answer:

$$R_{RL\text{ eff}} = \underline{0.5} \text{ k}\Omega \quad (0.4975... \text{ k}\Omega)$$



$$v_o = -\frac{R_f}{R_a/2} \cdot v_{cm} \quad \text{and} \quad v_{cm} = v_o + R_{RL} \cdot i_d$$

$$\Rightarrow R_{RL\text{ eff}} = \frac{v_{cm}}{i_d} = \frac{R_{RL}}{1 + 2 \frac{R_f}{R_a}} = \frac{100 \text{ k}\Omega}{1 + 200} \approx 0.5 \text{ k}\Omega$$

- d. [10 pts] Assume a displacement current of peak amplitude $i_d = 10 \mu\text{A}$ from 60 Hz line noise enters the body, and an ECG differential signal of peak amplitude $v_d = 1 \text{ mV}$ is generated by the heart. Find the amplitude of the resulting 60 Hz common-mode component of the voltage output $V_{out\ cm}$, and the corresponding signal-to-noise ratio SNR_{out} at the output.

Answer:

$$V_{out\ cm} = \underline{10. \text{ mV}}$$

$$\text{SNR}_{out} = \underline{2,000 \text{ (66 dB)}}$$

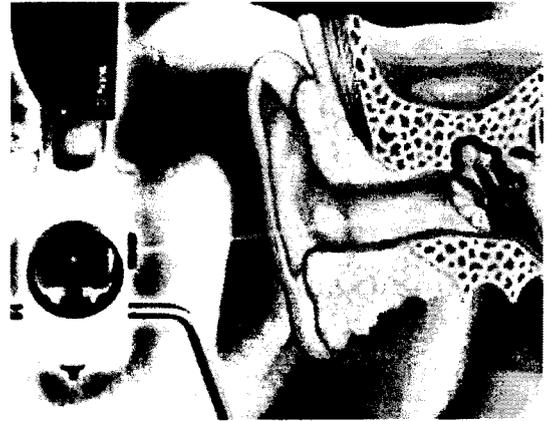
$$v_{cm} = R_{pl\ eff} \cdot i_d \approx 0.5 \text{ k}\Omega \cdot 10 \mu\text{A} = 5 \text{ mV}$$

$$v_{out\ cm} = A_c \cdot v_{cm} = \frac{A_d}{\text{CMRR}} \cdot v_{cm} = \frac{20,100}{10,000} \cdot 5 \text{ mV} \\ \approx 10. \text{ mV}$$

$$v_{out\ d} = A_d \cdot v_d = 20,100 \cdot 1 \text{ mV} \approx 20 \text{ V}$$

$$\text{SNR}_{out} = \frac{v_{out\ d}}{v_{out\ cm}} \approx \frac{20 \text{ V}}{10 \text{ mV}} = 2,000 \\ \text{(66 dB)}$$

3. [20 pts] An ENT doctor uses a laser Doppler vibrometer to measure the health of the eardrum. The 800 nm wavelength laser beam reflects off the eardrum and returns to a collector located at the same place as it was emitted. The doctor produces single-pitch sound at 20 kHz that causes the eardrum to vibrate sinusoidally with peak amplitude of 100 μm . Find the peak Doppler frequency shift recorded in the returning laser light. *HINT*: light wave frequency is given by $f = c/\lambda$ where $c = 3 \cdot 10^8$ m/s is the speed of light, and λ is the wavelength.



A laser Doppler vibrometer (Polytec) interrogating the eardrum

Doppler frequency shift:

$$\Delta f = f_R - f_S \approx \frac{v}{c} (\cos \theta_S + \cos \theta_R) f_S$$

Answer: $\Delta f_{\text{peak}} = \underline{10 \pi} \text{ MHz} \approx 31.4 \text{ MHz}$

$\cos \theta_S = \cos \theta_R = 1$ (perpendicular incidence on the eardrum)

$$\Rightarrow \Delta f_{\text{peak}} = \frac{v_{\text{peak}}}{c} \cdot 2 \cdot \frac{c}{\lambda} = 2 \frac{v_{\text{peak}}}{\lambda} \quad \lambda = 800 \text{ nm}$$

v_{peak} : $x(t) = A \sin(\omega t)$

$A = 100 \mu\text{m}$

$v(t) = A \omega \cos(\omega t)$

$\omega = 2\pi f_{\text{sound}}$

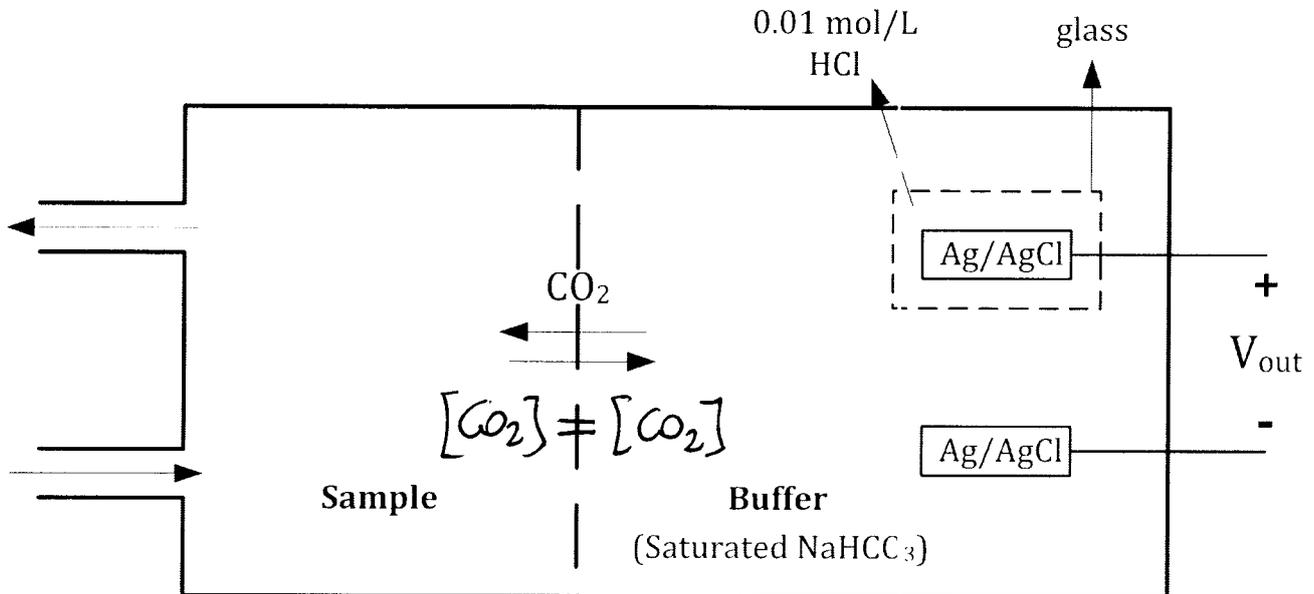
$v_{\text{peak}} = A \omega$

$f_{\text{sound}} = 20 \text{ kHz}$

$$\Rightarrow \Delta f_{\text{peak}} = 2 \frac{A \cdot 2\pi f_{\text{sound}}}{\lambda} = 2 \frac{100 \mu\text{m} \cdot 2\pi \cdot 20 \text{ kHz}}{800 \text{ nm}}$$

$$= 10,000 \pi \text{ kHz} = 10 \pi \text{ MHz}$$

4. [25 pts] A Severinghaus electrode for PCO_2 measurement is shown below. The inside compartment of the glass electrode contains a solution of 0.01 mol/L HCl, and the buffer contains a 0.1 mol/L solution of $NaHCO_3$. The two Ag/AgCl electrodes are identical with half potential $E_{Ag/AgCl} = 0.223$ mV. The membrane between the sample and buffer is permeable to CO_2 only, equalizing the concentration on both sides.



The Nernst potential for ion type A^n of valance n at room temperature is given by:

$$E_{\text{Nernst}} = \frac{60\text{mV}}{n} \cdot \log_{10} \frac{[A^n]_o}{[A^n]_i}$$

and the relationship between PCO_2 and $[H^+]$ and $[HCO_3^-]$ in the buffer at room temperature is given by:

$$PCO_2 = \frac{100 \text{ mmHg}}{(\text{mol/L})^2} [H^+] [HCO_3^-]$$

- a. [10 pts] Write V_{out} as a function of the pH of the buffer solution.

Answer: $V_{\text{out}} = \underline{60 \text{ mV} \cdot (2 - \text{pH})}$

$$V_{\text{out}} = E_{\text{Nernst (glass)}} = \frac{60 \text{ mV}}{1} \cdot \log_{10} \frac{[\text{H}^+]_{\text{buffer}}}{[\text{H}^+]_{\text{HCl}}}$$

$$= 60 \text{ mV} \cdot (-\text{pH} + 2)$$

↘ 0.01 mol/L HCl

b. [10 pts] Write V_{out} as a function of the PCO_2 of the sample solution.

Answer: $V_{\text{out}} = \frac{60 \text{ mV} \cdot (\log_{10} \text{PCO}_2 + 1)}{\text{with } \text{PCO}_2 \text{ in units mmHg}}$

$$\text{PCO}_2 = \frac{100 \text{ mmHg}}{(\text{mol/L})^2} [\text{H}^+] [\text{HCO}_3^-]$$

↘ = 0.1 mol/L (NaHCO₃)

$$\Rightarrow \log_{10} \text{PCO}_2 = \log_{10} \frac{10 \text{ mmHg}}{\text{mol/L}} + \log_{10} [\text{H}^+]$$

$$= 1 - \text{pH}$$

$$\Rightarrow V_{\text{out}} = 60 \text{ mV} \cdot (\log_{10} \text{PCO}_2 - 1 + 2)$$

$$= 60 \text{ mV} \cdot (\log_{10} \text{PCO}_2 + 1) \quad (\text{PCO}_2 \text{ in units mmHg})$$

c. [5 pts] For a given PCO_2 , does the output V_{out} depend on the flow rate of the blood sample in and out of the instrument? Does it depend on the temperature of the instrument? Explain.

Answer: V_{out} depends on sample flow rate: NO

V_{out} depends on temperature: YES

For a uniform blood sample, concentration is constant and does not depend on flow.

The reaction rates for $CO_2 + H_2O \rightleftharpoons H^+ + HCO_3^-$ depend on temperature, and the Nernst potential for the pH glass reference electrode depends on temperature as well.