BENG 186B Winter 2019

Quiz 1

Friday, January 25, 2019

***Name** (Last, First):* SOLUTIONS

- This quiz is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- You have 50 minutes to complete this quiz.

1. **[10 pts]** Circle the **best answer (only one answer per question)**:

   (a) [2.5 pts] Biomedical instruments with closed-loop feedback require the processing pipeline to have:

      i. high throughput.
      ii. low latency.
      iii. discrete-time dynamics.
      iv. all poles in the right-hand complex plane.

   (b) [2.5 pts] The Norton equivalent of an electrical circuit can be derived from:

      i. the open-circuit current and short-circuit voltage.
      ii. the Thévenin equivalent.
      iii. an equivalent current source in series with an impedance.
      iv. all of the above.

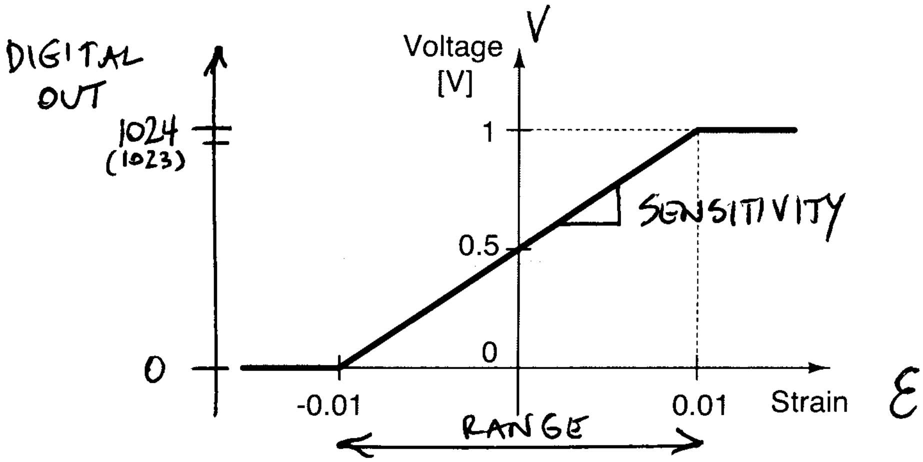
   (c) [2.5 pts] A piezoelectric sensor transduces:

      i. stress to voltage.
      ii. strain to charge.
      iii. strain to voltage.
      iv. all of the above.

   (d) [2.5 pts] A potentiometer is:

      i. a variable voltage divider.
      ii. a voltage sensor.
      iii. and instrument with zero output impedance.
      iv. all of the above.

2. **[30 pts]** You are given a biomedical instrument that measures strain and produces a digital reading on an output display. The instrument transduces the strain into a voltage, and digitizes this voltage by a 10-bit analog-to-digital converter (ADC). The transducer voltage as a function of strain is shown in the graph below, and the ADC full-scale voltage range is from 0 V to 1 V.



(a) [5 pts] Find the sensitivity of the strain-to-voltage transducer, and the range of strain over which it operates.

Sensitivity : $S = \frac{dV}{d\varepsilon} = \frac{1V}{0.02} = 50\ V$

Range : $[-0.01, 0.01]$ (0.02)

(b) [5 pts] Find the resolution of the instrument, and the range of strain over which it produces a valid digital reading.

Resolution : $\frac{\text{Range}}{2^{10}} = \frac{0.02}{1024} = 2\ 10^{-5}$

Same Range $[-0.01, 0.01]$ (0.02)

(c) [10 pts] You discover that the transducer for known strain values produces a voltage that on average is 0.05V lower than expected, and with a standard deviation of 0.01V. Find the relative accuracy and precision of the instrument.

Absolute:

$$\text{Accuracy} = \text{True} - \text{Mean} = \frac{0.05\text{V}}{5} = 0.001$$

$$\text{Precision} = \text{St. Dev.} = \frac{0.01\text{V}}{5} = 0.0002$$

Relative, to full scale:

$$\text{Accuracy} = \frac{\text{True} - \text{Mean}}{\text{Range}} = \frac{0.05\text{V}}{1\text{V}} = 5\%$$

$$\text{Precision} = \frac{\text{St. Dev}}{\text{Range}} = \frac{0.01\text{V}}{1\text{V}} = 1\%$$

(d) [10 pts] Now you are given two identical copies of the instrument. Describe how you would use these two to construct a better instrument that produces a consistent zero reading for zero strain independent of temperature and other environmental variations.

Subject the first copy to the strain at the input; subject the second copy to zero strain (no input connection)(*); and subtract the two digital readings to produce the output.

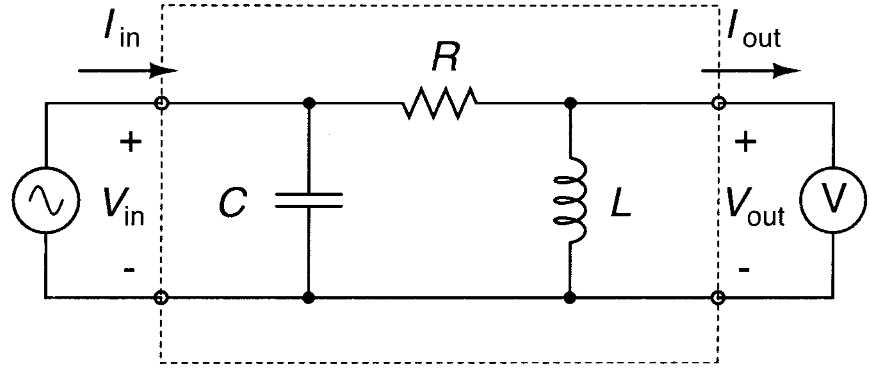
(*) OR: the opposing strain, if available

3. **[35 pts]** Consider the voltage-input, voltage-output filter circuit below.

VOLTAGE IN:

$0\,\Omega$

(SHORT)

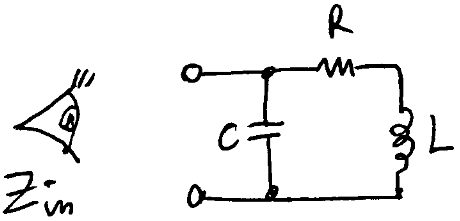


VOLTAGE OUT:

$\infty\,\Omega$

(OPEN)

(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$.

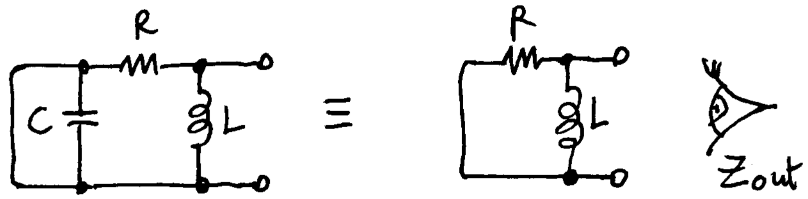


$$Z_{in}(j\omega) = \frac{1}{j\omega C} \,\Big\|\, (R + j\omega L)$$

$$= \frac{1}{j\omega C + \frac{1}{R + j\omega L}} = \frac{R + j\omega L}{(R + j\omega L)j\omega C + 1}$$

$$= \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC}$$

(b) [5 pts] Find the output impedance $Z_{out}(j\omega)$.

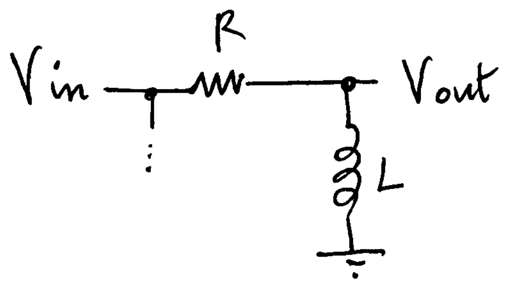


$$Z_{out}(j\omega) = j\omega L \parallel R$$

$$= \frac{1}{\frac{1}{j\omega L} + \frac{1}{R}}$$

$$= \frac{j\omega L}{1 + j\omega \frac{L}{R}}$$

(c) [5 pts] Find the transfer function $H(j\omega) = V_{out}(j\omega) \,/\, V_{in}(j\omega)$.



VOLTAGE DIVIDER

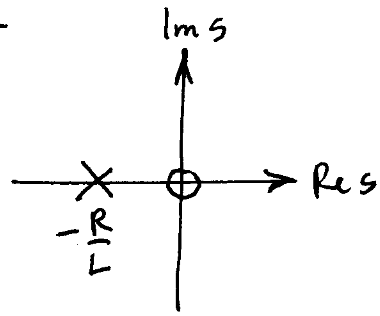
$$H(j\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R}}$$

HIGH-PASS

(d):

ZERO @ $s = 0$

POLE @ $s = -\frac{R}{L}$

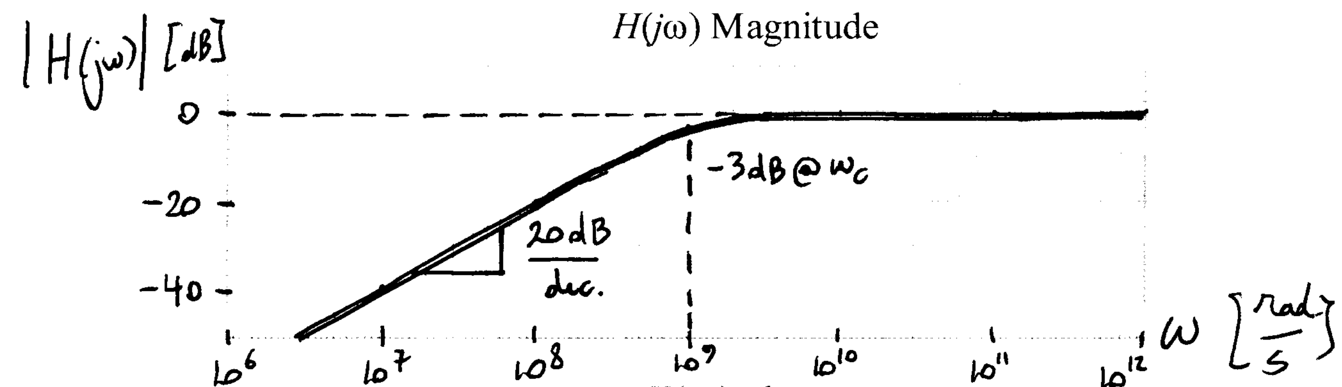


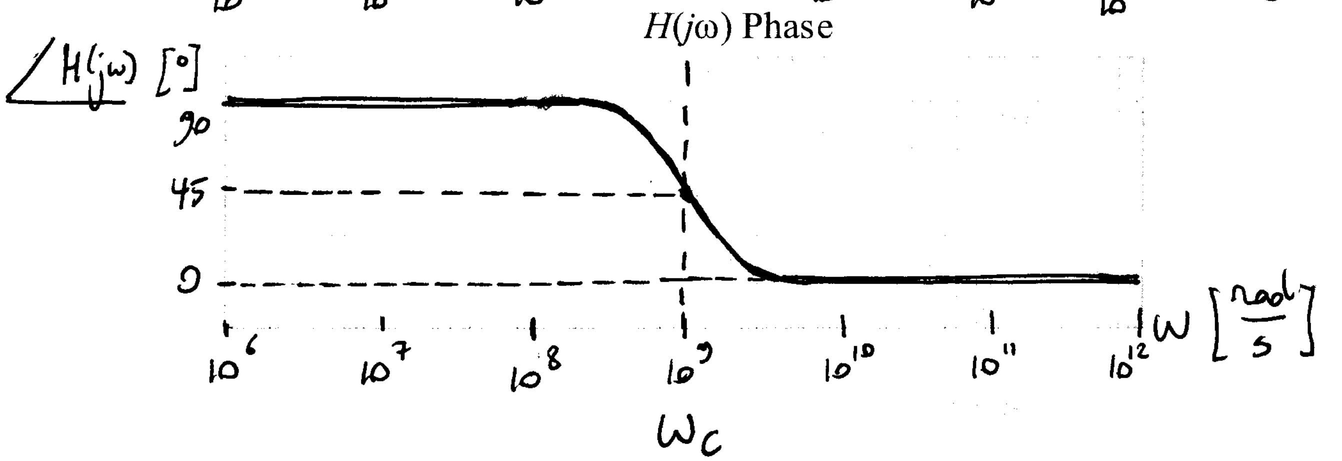
(d) [15 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C = 10$ nF, $R = 10$ kΩ, and $L = 10$ μH. Be sure to label the axes and indicate the units (rad/s, dB, and degrees).

HIGH-PASS with:

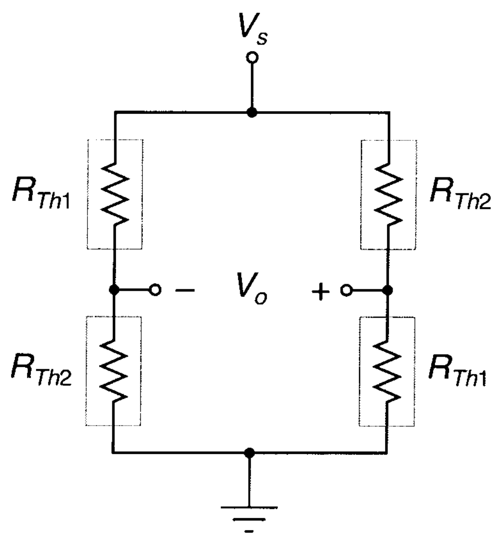
cut-off frequency : $\omega_c = \frac{R}{L} = \frac{10\,k\Omega}{10\,\mu H} = 10^9 \frac{rad}{s}$

in-band gain : $|H(\infty)| = 1 \quad (0\ dB)$





4. **[25 pts]** Consider the temperature-to-voltage transducer shown below, with constant supply voltage and two pairs of identical thermistors R_{Th1} and R_{Th2}.



$$
\begin{aligned}
R_{Th1} &= R_{nom1}\,(1+\alpha_1 T) \\
R_{Th2} &= R_{nom2}\,(1+\alpha_2 T)
\end{aligned}
$$

(a) [10 pts] Find the output voltage V_o as a function of temperature T, supply voltage V_s, nominal resistances R_{nom1} and R_{nom2}, and temperature coefficients α_1 and α_2.

$$V_o = \left(\frac{R_{Th1}}{R_{Th1}+R_{Th2}} - \frac{R_{Th2}}{R_{Th1}+R_{Th2}} \right) V_s$$

$$= \frac{R_{Th1} - R_{Th2}}{R_{Th1} + R_{Th2}} V_s$$

$$= \frac{(R_{nom1} - R_{nom2}) + (R_{nom1}\alpha_1 - R_{nom2}\alpha_2)T}{(R_{nom1} + R_{nom2}) + (R_{nom1}\alpha_1 + R_{nom2}\alpha_2)T} V_s$$

(b) [15 pts] Show that if $R_{nom1}\alpha_1 + R_{nom2}\alpha_2 = 0$ the transducer is linear. Find its sensitivity S, and offset temperature T_{off} at which the output voltage is zero.

$$R_{nom1}\alpha_1 + R_{nom2}\alpha_2 = 0 \Rightarrow$$

$$V_o = \frac{V_s}{R_{nom1}+R_{nom2}}\Big((R_{nom1}-R_{nom2})+(R_{nom1}\alpha_1-R_{nom2}\alpha_2)T\Big)$$

LINEAR in T

$$\text{Sensitivity } S = \frac{dV_o}{dT} = \frac{R_{nom1}\alpha_1 - R_{nom2}\alpha_2}{R_{nom1}+R_{nom2}} V_s$$

Offset T_{off} such that $(R_{nom1}-R_{nom2})+(R_{nom1}\alpha_1-R_{nom2}\alpha_2)T_{off}=0$

$$\Rightarrow T_{off} = -\frac{R_{nom1}-R_{nom2}}{R_{nom1}\alpha_1 - R_{nom2}\alpha_2}$$