

# BENG 186B Winter 2020

## Quiz 1

Friday, January 24, 2020

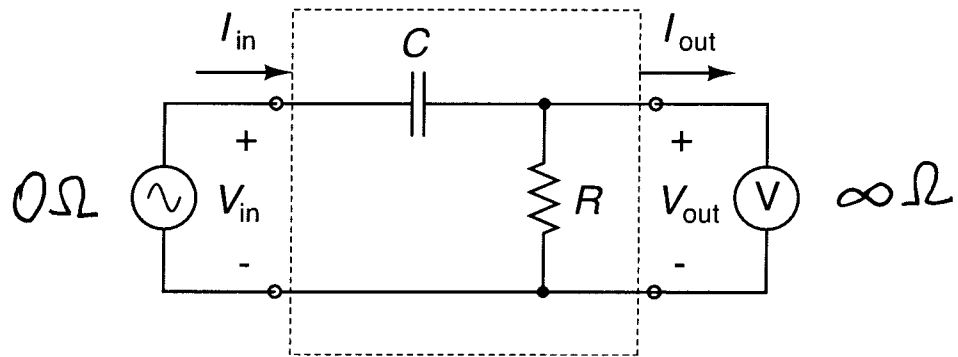
*Name (Last, First):* SOLUTIONS

- This quiz is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- The quiz takes 50 minutes to complete.

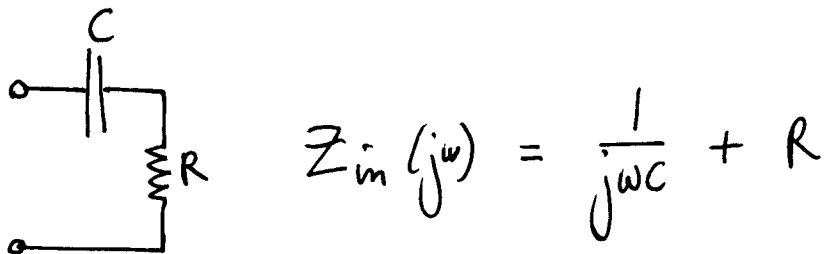
1. [15 pts] Circle the best answer (only one answer per question):

- (a) [3 pts] Averaging readings from multiple measurements of the same measur- and improves its:
- i. sensitivity.
  - ii. accuracy.
  - iii. precision.
  - iv. none of the above.
- (b) [3 pts] The transfer function of an underdamped ( $\zeta < 1$ ) second-order low-pass filter has:
- i. two complex conjugate poles.
  - ii. two identical real poles.
  - iii. two different real poles.
  - iv. one real pole and one imaginary pole.
- (c) [3 pts] Measuring the open-circuit voltage and short-circuit current of a circuit element determines its:
- i. Thévenin equivalent.
  - ii. Norton equivalent.
  - iii. equivalent impedance.
  - iv. all of the above.
- (d) [3 pts] The sensitivity of a piezoresistive stress transducer depends on the:
- i. gauge factor.
  - ii. Young's modulus.
  - iii. nominal resistance.
  - iv. all of the above.
- (e) [3 pts] The sensitivity of a capacitive displacement transducer can be improved by:
- i. decreasing the area of the two plates
  - ii. decreasing the distance between the two plates.
  - iii. lowering the dielectric constant of the insulator between the plates.
  - iv. all of the above.

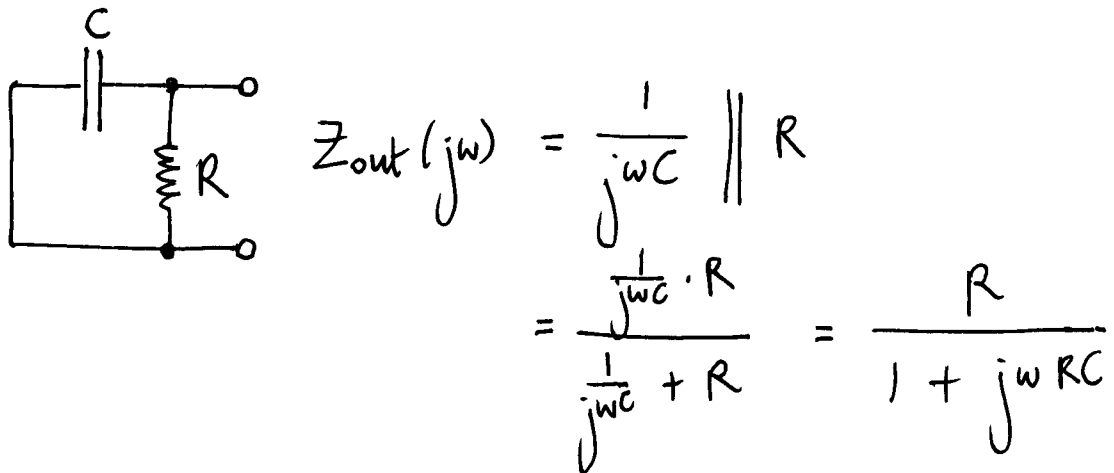
2. [45 pts] Consider the voltage-input, voltage-output filter circuit below.



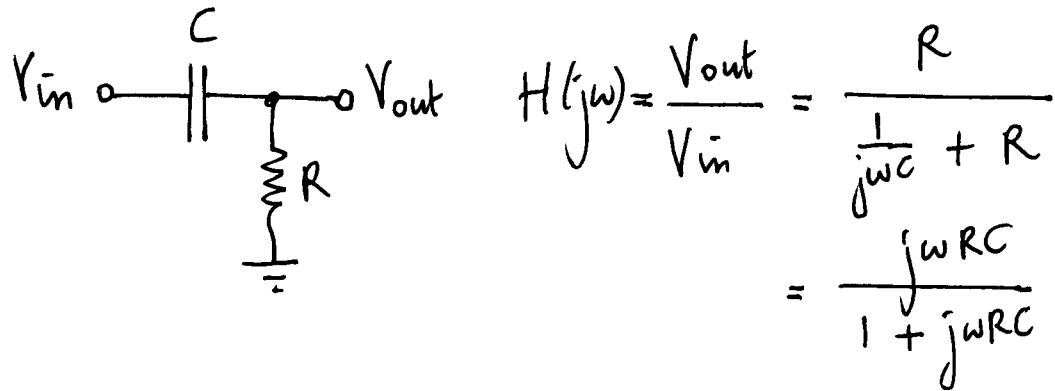
(a) [5 pts] Find the input impedance  $Z_{in}(j\omega)$ .



(b) [10 pts] Find the output impedance  $Z_{out}(j\omega)$ .



(c) [10 pts] Find the transfer function  $H(j\omega) = V_{out}(j\omega) / V_{in}(j\omega)$ .



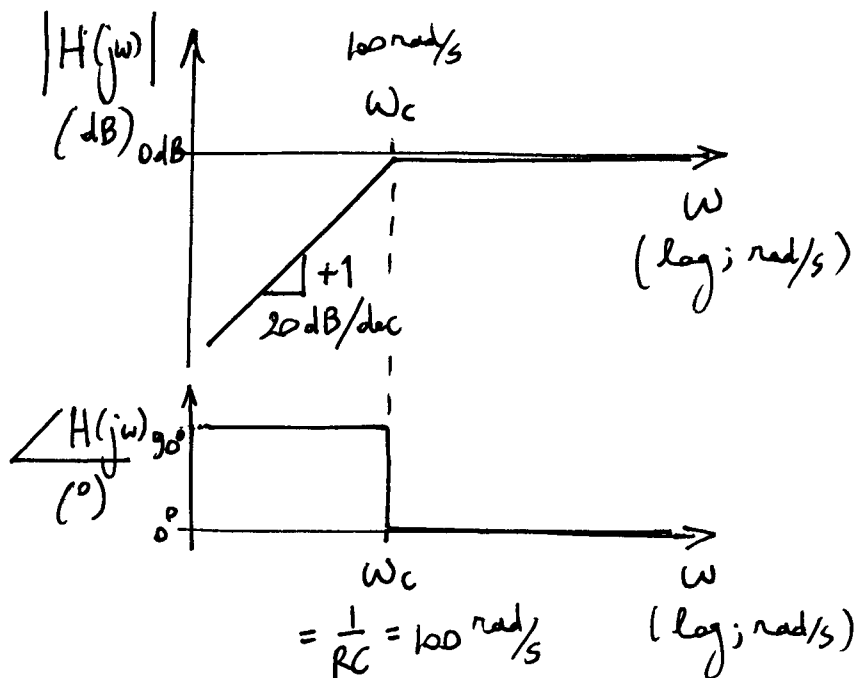
(d) [10 pts] Sketch the Bode plot of the transfer function  $H(j\omega)$  for  $C = 100$  nF and  $R = 100$  k $\Omega$ . Be sure to label the axes and indicate the units (e.g., rad/s, dB, and degrees). What type of filter is this?

$$H(s) = \frac{sRC}{1 + sRC}$$

zero @  $s = 0$

pole @  $s = -\frac{1}{RC} = -\frac{1}{10^{-7} \cdot 10^5 \text{ s}} = -100 \frac{\text{rad}}{\text{s}}$

$\underbrace{\hspace{10em}}_{\omega_c}$



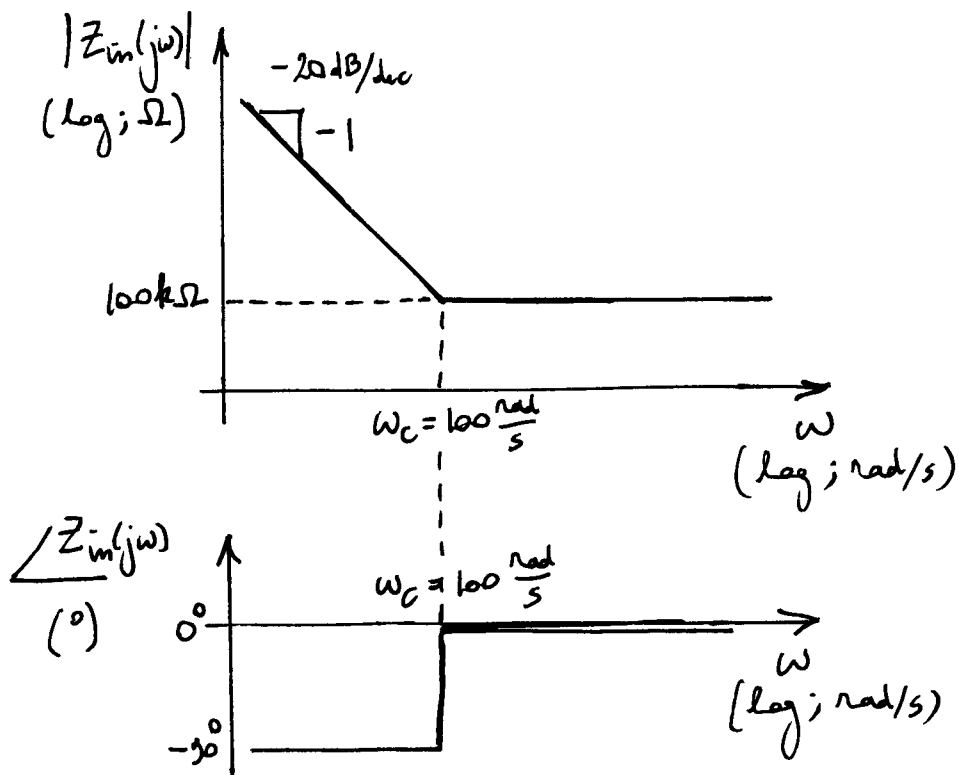
HIGHPASS FILTER

- (e) [10 pts] Sketch the Bode plot of the input impedance  $Z_{in}(j\omega)$  for the same values of  $C$  and  $R$  as in (d). Again be sure to label the axes and indicate the units (e.g., rad/s, dB $\Omega$ , and degrees).

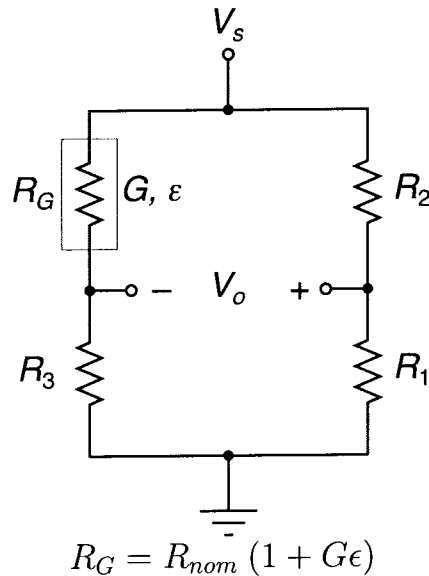
$$Z_{in}(s) = \frac{1}{sC} + R = \frac{1 + sRC}{sC}$$

$$\text{zero @ } s = -\frac{1}{RC} = -100 \frac{\text{rad}}{\text{s}}$$

$$\text{pole @ } s = 0$$



3. [40 pts] Consider the strain transducer below, with constant supply voltage  $V_s = 1$  V, three resistors with identical resistances  $R_1 = R_2 = R_3 = R$ , and a strain gauge  $R_G$  with nominal resistance  $R_{nom} = 100$  k $\Omega$  and gauge factor  $G = 100$ . The transducer produces a differential output voltage  $V_o$  in response to a strain  $\epsilon$  applied to the strain gauge.



- (a) [10 pts] Find the value of resistance  $R$  that maximizes the sensitivity of the transducer. Find this maximum sensitivity. *Hint:* you may assume that the applied strain is sufficiently small in magnitude so that  $|G\epsilon| \ll 1$ .

$$V_o = \left( \frac{R}{R+R} - \frac{R}{R+R_G} \right) V_s$$

$$= \left( \frac{1}{2} - \frac{1}{1 + \frac{R_{nom}}{R}(1+G\epsilon)} \right) V_s$$

Sensitivity:  $S = \frac{dV_o}{d\epsilon} = \frac{\frac{R_{nom}}{R} G V_s}{\left(1 + \frac{R_{nom}}{R}\right)^2}$  maximum for  $R = R_{nom}$   
 (@  $\epsilon = 0$ )

@ maximum:  $S = \frac{1}{4} G V_s = 25$  V

(b) [5 pts] Find the offset of the transducer.

$$\textcircled{a} \quad \varepsilon = 0 : V_o = \left(\frac{1}{2} - \frac{1}{2}\right)V_s = 0$$

$$\text{so: offset} = 0$$

- (c) [10 pts] Due to mismatch in manufacturing, the actual  $R_1$  is 1% smaller than expected, and the actual  $R_2$  is 1% larger than expected. Find the accuracy of the strain measurement.

$$\textcircled{a} \quad \epsilon = 0 : \quad V_o = \left( \frac{0.99R}{0.99R + 1.01R} - \frac{1}{2} \right) \cdot 1V = -0.005V$$

(TRUE  $\epsilon$ )

$$\text{MEASURED } \epsilon : \quad \frac{V_o}{S} = \frac{-0.005V}{25V} = -2 \cdot 10^{-4}$$

$$\Rightarrow \text{ACCURACY : } \left| \text{MEASURED} - \text{TRUE} \right| = 2 \cdot 10^{-4}$$

- (d) [10 pts] An 8-bit analog-to-digital converter (ADC) is used to digitize the voltage output  $V_o$  for a digital reading of the strain  $\epsilon$ . The full-scale voltage range of the ADC is from 0.4 V to 0.6 V. Find the resolution of the strain measurement.

$$\text{Resolution} = \frac{\text{Range} / 2^m}{S} = \frac{0.2V / 256}{25V} = 3 \cdot 10^{-5}$$

- (e) [5 pts] You observe that the transducer is sensitive to temperature. What simple change could you make to the design to eliminate this temperature dependence? *Hint:* you may assume that all resistances  $R$  have identical temperature coefficient, and also that strain gauge nominal resistances have identical temperature coefficient, but these two are different.

Replace  $R_3$  with an identical strain gauge  $R_G$ , but with NO strain applied:

