BENG 186B Winter 2021

Quiz 1

Friday, January 22, 2021

***Name** (Last, First):* SOLUTIONS

- This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due January 22, 2021 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[15 pts]** Circle the **best answer (only one answer per question)**:

   (a) [3 pts] Calibration of a bioinstrument using a known reference for the measurand improves its:

      i. sensitivity.
      ii. accuracy.
      iii. precision.
      iv. none of the above.

   (b) [3 pts] A second-order system with two distinct negative real poles is:

      i. underdamped.
      ii. critically damped.
      iii. overdamped.
      iv. all of the above.

   (c) [3 pts] The power dissipated by a circuit element is zero when its:

      i. current is zero.
      ii. voltage is zero.
      iii. voltage and current are 90 degrees out of phase.
      iv. all of the above.

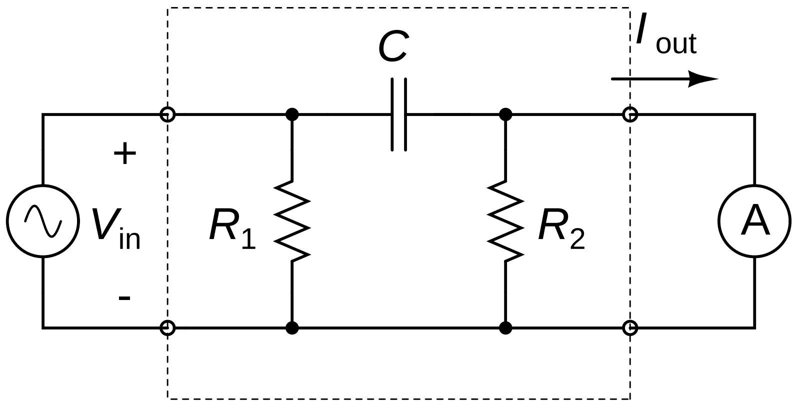
   (d) [3 pts] A piezoelectric sensor:

      i. transduces stress to charge.
      ii. changes its resistance under strain.
      iii. requires a Wheatstone bridge for accurate measurement.
      iv. none of the above.

   (e) [3 pts] A linear variable differential transformer:

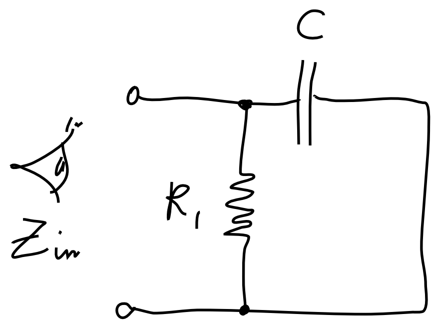
      i. is a type of inductive displacement sensor.
      ii. offers twice higher sensitivity than a single variable transformer.
      iii. has zero offset.
      iv. all of the above.

2. **[40 pts]** Consider the **voltage-input, current-output** filter circuit below.



(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$. *Hint*: consider an ideal load (current meter) at the output.

The ideal load (current meter) at the output represents a short circuit, shorting R2.

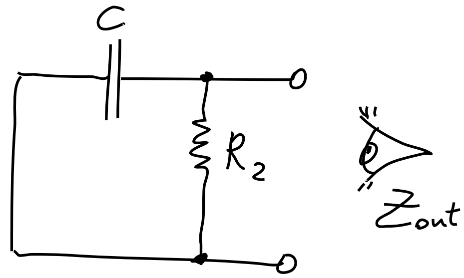


$$Z_{in}(j\omega) = R_1 \parallel \frac{1}{j\omega C} = \frac{R_1 \frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}}$$

$$= \frac{R_1}{1 + j\omega R_1 C}$$

(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.

Similarly, the ideal source (voltage source) at the input represents a short circuit, shorting R1.



$$Z_{out}(j\omega) = R_2 \parallel \frac{1}{j\omega C} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega R_2 C}$$

(c) [10 pts] Find the transfer function $H(j\omega) = I_{out}(j\omega) / V_{in}(j\omega)$.

All current through the capacitor flows through the current meter, since its impedance is zero (current flows along the path of least resistance).

The voltage across the capacitor is Vin.

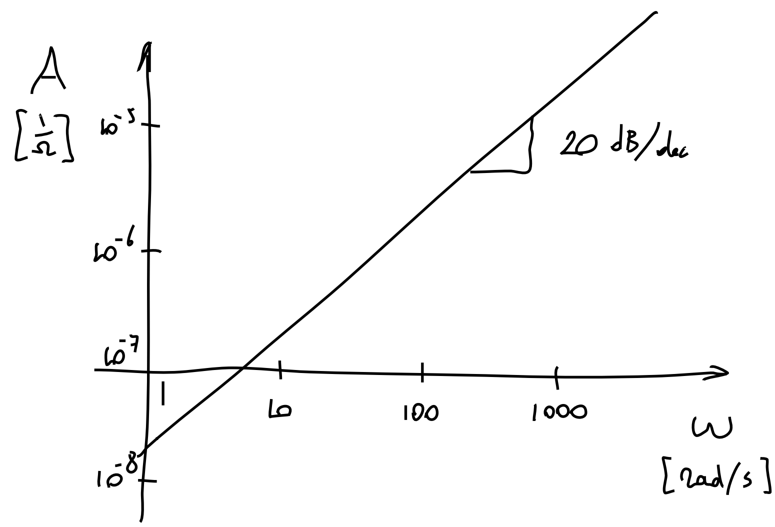
$$\Rightarrow \quad I_{out} = j\omega C \cdot V_{in}$$

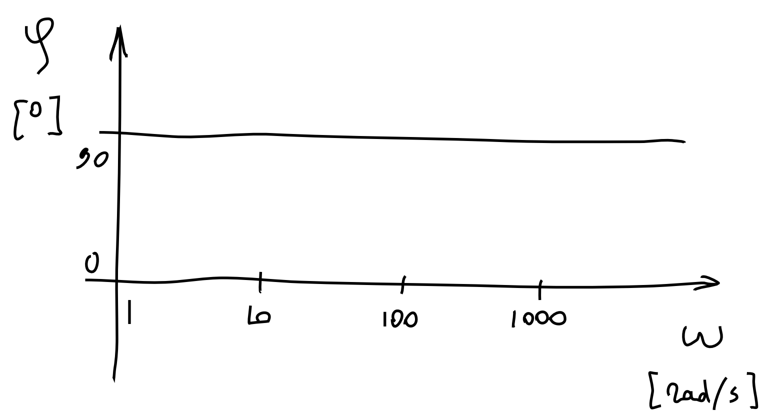
$$H(j\omega) = \frac{I_{out}(j\omega)}{V_{in}(j\omega)} = j\omega C$$

(d) [10 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C = 10$ nF, and $R_1 = R_2 = 10$ kΩ. Be sure to label the axes and indicate the units (rad/s, A/V or 1/Ω, and degrees).

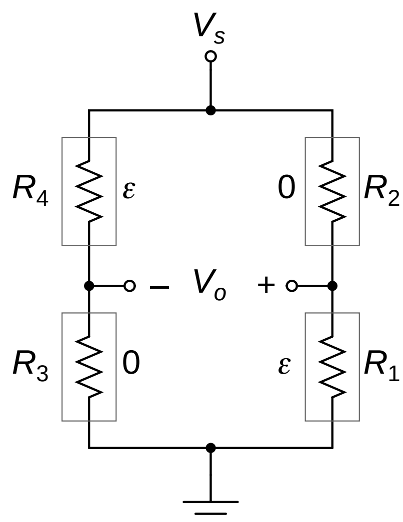
$$H(j\omega) = j\omega C \quad ; \qquad \text{single zero at } s = 0$$

$$\begin{cases} A(\omega) = \omega C \\ \varphi(\omega) = 90^\circ \end{cases}$$





3. **[45 pts]** Consider the stress transducer below, with constant supply voltage $V_s = 5$ V, and four strain gauges R_1, R_2, R_3 and R_4 all with identical nominal resistance $R_{nom} = 10$ kΩ, gauge factor $G = 20$, and Young's modulus $E = 100$ kPa. The transducer produces a differential output voltage V_o in response to stress σ applied to two of the strain gauges R_1 and R_4, whereas the other two gauges R_2 and R_3 are not subjected to any stress.



$$R_1 = R_4 = R_{nom}\,(1 + G\,\epsilon)$$
$$R_2 = R_3 = R_{nom}$$

(a) [5 pts] Find the output voltage V_o as a function of stress σ.

$$V_o = \left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) V_s$$

$$= \left(\frac{R_{nom}(1 + G\frac{\sigma}{E})}{R_{nom}(1 + G\frac{\sigma}{E}) + R_{nom}} - \frac{R_{nom}}{R_{nom}(1 + G\frac{\sigma}{E}) + R_{nom}} \right) V_s$$

$$= \left(\frac{\cancel{1} + G\frac{\sigma}{E}}{2 + G\frac{\sigma}{E}} - \frac{\cancel{1}}{2 + G\frac{\sigma}{E}} \right) V_s = \frac{\frac{G}{E} V_s \sigma}{2 + \frac{G}{E}\sigma}$$

(b) [10 pts] Find the sensitivity and offset of the stress transducer. You may assume that the applied stress is sufficiently small in magnitude such that $|\frac{G}{E}\sigma| \ll 1$.

Zero offset: $V_o = \dfrac{\frac{G}{E} V_s \sigma}{2 + \frac{G}{E}\sigma} = 0$ for $\sigma = 0$

Sensitivity for small σ: $S = \dfrac{dV_o}{d\sigma} \approx \dfrac{1}{2}\dfrac{G}{E} V_s = 0.5 \dfrac{V}{kPa}$

$2 + \frac{G}{E}\sigma \approx 2$

(c) [10 pts] Find the number of bits and voltage range needed in analog-to-digital conversion of the transducer output in order to digitize the stress signal σ with 1 Pa resolution over a range from -500 Pa to +500 Pa.

Range: $\sigma_{min} = -500\ \text{Pa} \longrightarrow V_{o\,min} = -0.5\ \text{kPa} \cdot 0.5 \frac{V}{\text{kPa}} = -0.25\ V$

$\sigma_{max} = +500\ \text{Pa} \longrightarrow V_{o\,max} = +0.5\ \text{kPa} \cdot 0.5 \frac{V}{\text{kPa}} = +0.25\ V$

Resolution: $\Delta\sigma = 1\ \text{Pa} \longrightarrow \Delta\sigma = 2^{-n}\left(\sigma_{max} - \sigma_{min}\right)$

number of bits: $n \geq \log_2\left(\frac{\sigma_{max} - \sigma_{min}}{\Delta\sigma}\right) \approx 10$

(d) [10 pts] Why is it helpful to have four strain gauges, even though only two of these are actually subjected to strain? Explain what happens to the temperature sensitivity and accuracy of stress measurement if the gauges R_2 and R_3 were replaced with plain resistors.

Nominal resistance and temperature coefficient are matched across strain gauges, but not between strain gauges and plain resistors.

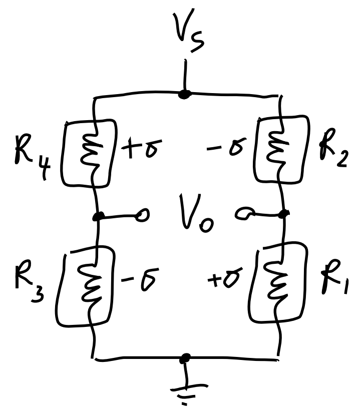
$$R_1 = R_4 = R_{nom,G}\left(1 + \alpha_G \Delta T\right)$$
$$R_2 = R_3 = R_{nom,R}\left(1 + \alpha_R \Delta T\right)$$

$R_{nom,G} \neq R_{nom,R} \Rightarrow$ Accuracy loss due to non-zero offset

$\alpha_G \neq \alpha_R \Rightarrow$ Non-zero temperature sensitivity

(e) [10 pts] Now assume that in addition to the stress $+\sigma$, you have its complement $-\sigma$ available for measurement. Show how to modify the setup of the transducer with four strain gauges to make use of the differential stress in order to increase the sensitivity of the stress transducer, while improving the accuracy of stress measurement by improving its linearity.

Standard double-differential strain gauge Wheatstone bridge (Lecture 3 notes):



$$R_2 = R_3 = R_{nom}\left(1 - G\frac{\sigma}{E}\right)$$

$$R_1 = R_4 = R_{nom}\left(1 + G\frac{\sigma}{E}\right)$$

$$V_o = \left(\frac{\cancel{R_{nom}}(\cancel{1} + G\frac{\sigma}{E})}{\cancel{R_{nom}}(1 + \cancel{G\frac{\sigma}{E}}) + \cancel{R_{nom}}(1 - \cancel{G\frac{\sigma}{E}})} - \frac{\cancel{R_{nom}}(\cancel{1} - G\frac{\sigma}{E})}{\cancel{R_{nom}}(1 + \cancel{G\frac{\sigma}{E}}) + \cancel{R_{nom}}(1 - \cancel{G\frac{\sigma}{E}})}\right) V_s$$

$$= \left(\frac{1}{2} G\frac{\sigma}{E} - \left(-\frac{1}{2} G\frac{\sigma}{E}\right)\right) V_s = \frac{G}{E} V_s \cdot \sigma$$

Twice larger sensitivity $\quad \frac{G}{E} V_s = 1 \frac{V}{kPa}$

Perfect linearity