BENG 186B Winter 2022

Quiz 1

Friday, January 21, 2022

Name (Last, First): SOLUTIONS

• This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.

• The quiz is due January 21, 2022 at 11:59pm, over Canvas Gradescope. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.

• There are 3 problems. Points for each problem are given in [brackets]. There are 100 points total.
1. **[15 pts]** Circle the best answer (only one answer per question):

(a) **[3 pts]** Precision of a bioinstrument can be improved by:
   i. lowering the measurement noise.
   ii. averaging multiple readings.
   iii. increasing the signal amplitude at its input.
   iv. all of the above.

(b) **[3 pts]** To accurately digitize a 1 kHz bandwidth biosignal the sampling rate needs to be at least:
   i. 500 samples per second.
   ii. 1,000 samples per second.
   iii. 2,000 samples per second.
   iv. Whatever, it doesn’t matter.

(c) **[3 pts]** Cascading two first-order lowpass filter sections with different cut-off frequencies produces a second-order lowpass filter that is:
   i. underdamped.
   ii. critically damped.
   iii. overdamped. (two real, different poles)
   iv. none of the above.

(d) **[3 pts]** A thermistor is a resistor with a temperature coefficient that is:
   i. always positive.
   ii. relatively large.
   iii. temperature independent.
   iv. all of the above.

(e) **[3 pts]** The sensitivity of an inductive displacement transducer can be improved by:
   i. choosing a core with larger magnetic permeability $\mu$.
   ii. lowering the number of windings in the coil.
   iii. bringing a permanent magnet in close proximity.
   iv. all of the above.
2. [40 pts] Consider the current-input, current-output filter circuit below. You may assume an ideal current source for $I_{in}$, and an ideal ammeter for $I_{out}$.

![Circuit Diagram]

(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$.

![Modified Circuit Diagram]

$$Z_{in} = \frac{1}{j\omega C} \parallel R_L = \frac{1}{\frac{1}{j\omega C} + R_L} = \frac{R_L}{1 + j\omega R_L C}$$
(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.

\[ Z_{out} = R_2 \]

(c) [10 pts] Find the transfer function $H(j\omega) = I_{out}(j\omega) / I_{in}(j\omega)$. What are the units? Does it depend on frequency?

\[ \Rightarrow \quad I_{out} = I_{in} \quad \Rightarrow \quad H(j\omega) = 1 \]

Dimensionless & Constant over frequency
(d) [10 pts] Sketch the Bode plot of the input impedance $Z_{in}(j\omega)$ for $C = 10$ nF, $R_1 = 100$ kΩ, and $R_2 = 1$ MΩ. Be sure to label the axes and indicate the units (e.g., rad/s, dBΩ, and degrees).

$$Z_{in}(j\omega) = \frac{R_1}{1 + j\omega R_1 C} : \text{1 pole @ } s = j\omega = -\frac{1}{R_1 C} = -10^3 \frac{\text{rad}}{s}$$

\[ A \ [\text{dBΩ}] \]

\[ 60 \text{dBΩ} \]

\[ 1 \text{MΩ} \]

\[ \frac{1}{R_1 C} = 10^3 \frac{\text{rad}}{s} \]

\[ -20 \text{dB/dec} \]

\[ \omega \ [\log] \]

\[ \phi \ [\degree] \]

\[ 10^3 \frac{\text{rad}}{s} \]

\[ -90^\circ \]

\[ -45^\circ \]
3. [45 pts] Consider the stress transducer below, with constant supply voltage $V_s = 3 \text{ V}$, two identical current sources $I_{ref} = 10 \mu \text{A}$, a resistor $R$, and a strain gauge $R_G$ with nominal resistance $R_{\text{nom}} = 100 \text{ k}\Omega$, gauge factor $G = 100$, and Young’s modulus $E = 100 \text{ kPa}$. The transducer produces a differential output voltage $V_\text{o}$ in response to a stress $\sigma$ applied to the strain gauge.

\[
R_G = R_{\text{nom}} (1 + G \epsilon)
\]

\[
\sigma = E \epsilon
\]

(a) [10 pts] Find the output voltage $V_\text{o}$ as a function of the applied stress $\sigma$. Does it depend on the supply voltage, and why?

\[
V_\text{o} = R_G \cdot \text{Inj} - R \cdot \text{Inj}
\]

\[
= R_{\text{nom}} (1 + \frac{G}{E} \sigma) \cdot \text{Inj} - R \cdot \text{Inj}
\]

*INDEPENDENT of supply voltage $V_s$ for ideal current sources $\text{Inj}$*
\[ V_0 = (R_{\text{nom}} - R) \cdot I_{\text{ref}} + R_{\text{nom}} \cdot I_{\text{ref}} \cdot \frac{G}{E} \cdot \sigma \]

(b) [5 pts] Find the value of resistance \( R \) that nulls the offset of the transducer. Use this value of \( R \) for the remainder of this problem.

\[ 3(b) : \quad V_{\text{off}} = 0 \quad \text{for} \quad R = R_{\text{nom}} = 100 \ \text{k}\Omega \]

(c) [5 pts] Find the sensitivity of the transducer.

\[ 3(c) : \quad S = R_{\text{nom}} \cdot I_{\text{ref}} \cdot \frac{G}{E} = 100 \ \text{k}\Omega \cdot 10 \mu\text{A} \cdot \frac{100}{100 \ \text{k}\Omega} \]

\[ = 1 \ \text{V/\text{kPa}} \]
(d) [10 pts] Due to mismatch in manufacturing of the instrument, the left current source $I_{ref}$ is actually 100 nA lower than expected, whereas the right current source $I_{ref}$ is actually 100 nA higher than expected. Find the absolute accuracy of stress measurement by the instrument at $\sigma = 0$, and at $\sigma = 1$ kPa.

$$V_{0,\text{actual}} = R_{\text{nom}} \left( 1 + \frac{G}{E} \sigma \right) (I_{\text{ref}} + 100\text{nA}) - R_{\text{nom}} (I_{\text{ref}} - 100\text{nA})$$

$$V_{0,\text{ideal}} = R_{\text{nom}} \left( 1 + \frac{G}{E} \sigma \right) I_{\text{ref}} - R_{\text{nom}} I_{\text{ref}}$$

$$\text{Accuracy} (\sigma) = \frac{\text{Accuracy} (V_0)}{S}$$

$$= \frac{1}{S} \cdot \left| V_{0,\text{actual}} - V_{0,\text{ideal}} \right|$$

$$= \frac{1}{S} \cdot \left| R_{\text{nom}} \left( 1 + \frac{G}{E} \sigma \right) \cdot 100\text{nA} - R_{\text{nom}} \cdot (-100\text{nA}) \right|$$

$$= \frac{R_{\text{nom}} \cdot 100\text{nA}}{S} \cdot \left| 2 + \frac{G}{E} \sigma \right|$$

$$= 10 \text{ Pa} \cdot \left| 2 + \frac{\sigma}{1\text{ kPa}} \right|$$

\[ \sigma = 0 \quad \Rightarrow \quad \text{Accuracy} (\sigma) = 10 \text{ Pa} \cdot 2 = 20 \text{ Pa} \]

\[ \sigma = 1 \text{ kPa} \quad \Rightarrow \quad \text{Accuracy} (\sigma) = 10 \text{ Pa} \cdot \left( 2 + \frac{1\text{ kPa}}{1\text{ kPa}} \right) = 30 \text{ Pa} \]
(c) [10 pts] An analog-to-digital converter (ADC) is used to digitize the voltage output \( V_o \) for a digital reading of the stress \( \sigma \). Find the full-scale voltage range and the number of bits of the ADC required for a full-scale stress range from 0 to 1 kPa at a resolution of 1 Pa.

\[
\begin{align*}
\sigma = 0 & \implies V_o = V_{off} = 0 \\
\sigma = 1 \text{ kPa} & \implies V_o = V_{off} + \frac{1 \text{ kPa}}{1 \text{ kPa/V}} = 1 \text{ V}
\end{align*}
\]

\( \implies \) ADC voltage range : 0 to 1 V

\[
\begin{align*}
\Delta \sigma = 1 \text{ Pa} & \implies \Delta V_o = \frac{1 \text{ Pa}}{1 \text{ kPa/V}} = 1 \text{ mV}
\end{align*}
\]

\( \implies \) ADC number of bits = \( \log_2 \frac{1 \text{ V}}{1 \text{ mV}} = 10 \) bits
(f) [5 pts] You observe that the transducer is subject to a drifting offset due to variations in temperature. What simple change could you make to the design in order to eliminate this temperature dependence? *Hint:* you may assume that all identical components, such as the two current sources, have identical temperature coefficients.

Replace $R$ with a matching strain gage $R_G$, but with zero stress.