BENG 186B Winter 2022

Quiz 1

Friday, January 21, 2022

***Name** (Last, First):* SOLUTIONS

- This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due January 21, 2022 at 11:59pm, over Canvas Gradescope. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[15 pts]** Circle the **best answer (only one answer per question)**:

   (a) [3 pts] Precision of a bioinstrument can be improved by:

      i. lowering the measurement noise.
      ii. averaging multiple readings.
      iii. increasing the signal amplitude at its input.
      iv. all of the above.

   (b) [3 pts] To accurately digitize a 1 kHz bandwidth biosignal the sampling rate needs to be at least:

      i. 500 samples per second.
      ii. 1,000 samples per second.
      iii. 2,000 samples per second.
      iv. Whatever, it doesn't matter.

   (c) [3 pts] Cascading two first-order lowpass filter sections with different cut-off frequencies produces a second-order lowpass filter that is:

      i. underdamped.
      ii. critically damped.
      iii. overdamped. (two real, different poles)
      iv. none of the above.

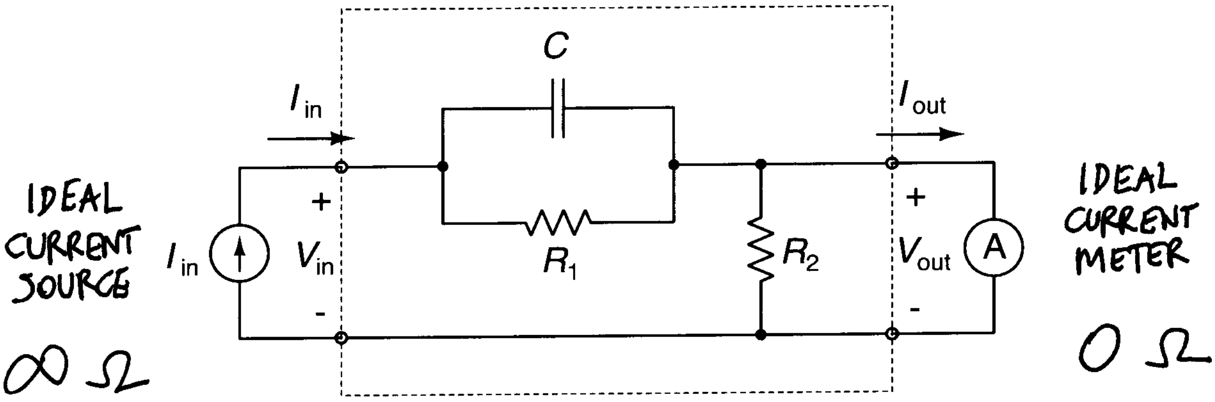
   (d) [3 pts] A thermistor is a resistor with a temperature coefficient that is:

      i. always positive.
      ii. relatively large.
      iii. temperature independent.
      iv. all of the above.

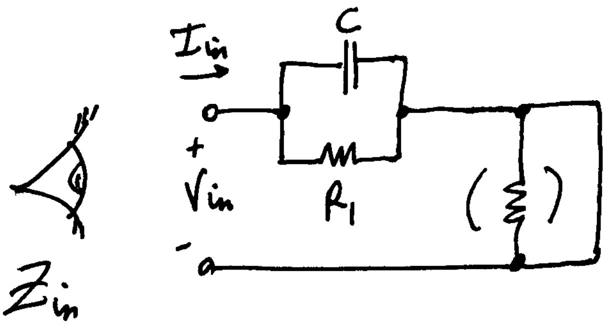
   (e) [3 pts] The sensitivity of an inductive displacement transducer can be improved by:

      i. choosing a core with larger magnetic permeability μ.
      ii. lowering the number of windings in the coil.
      iii. bringing a permanent magnet in close proximity.
      iv. all of the above.

2. **[40 pts]** Consider the *current-input*, *current-output* filter circuit below. You may assume an ideal current source for I_{in}, and an ideal ammeter for I_{out}.



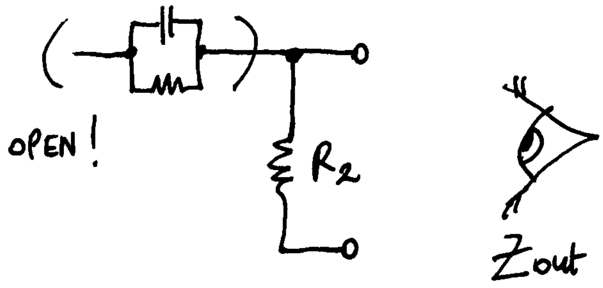
(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$.



output shorted

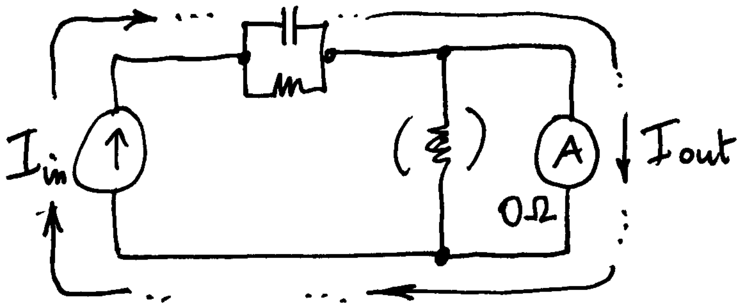
$$Z_{in} = \frac{1}{j\omega C} \parallel R_1 = \frac{\frac{1}{j\omega C} \cdot R_1}{\frac{1}{j\omega C} + R_1} = \frac{R_1}{1 + j\omega R_1 C}$$

(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.



$Z_{out} = R_2$

(c) [10 pts] Find the transfer function $H(j\omega) = I_{out}(j\omega) \,/\, I_{in}(j\omega)$. What are the units? Does it depend on frequency?



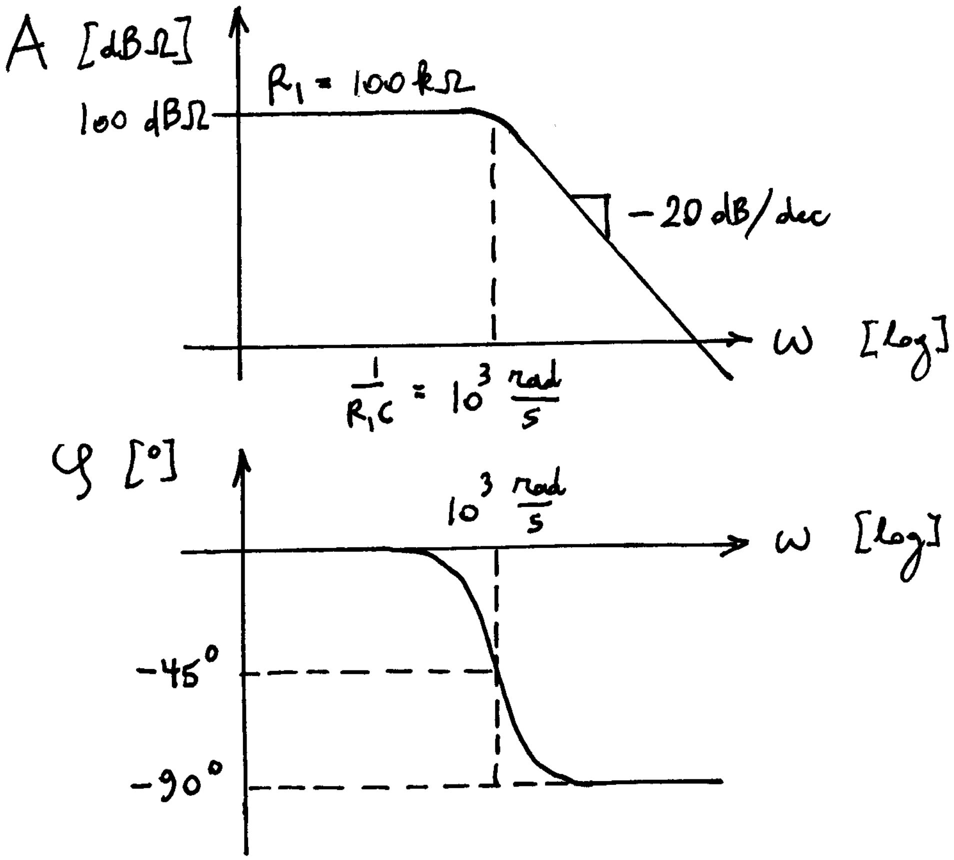
Current flows through the path of least resistance : 0 Ω

$\Rightarrow \quad I_{out} = I_{in} \quad \Rightarrow \quad H(j\omega) = 1$

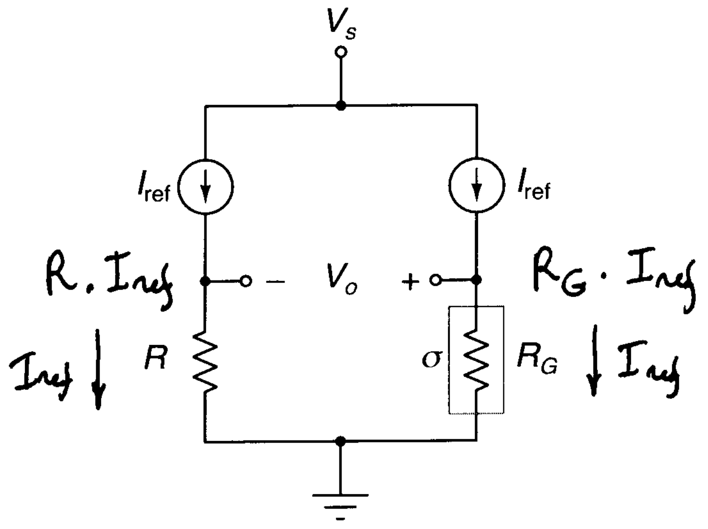
Dimensionless & Constant over frequency

(d) [10 pts] Sketch the Bode plot of the input impedance $Z_{in}(j\omega)$ for $C = 10$ nF, $R_1 = 100$ kΩ, and $R_2 = 1$ MΩ. Be sure to label the axes and indicate the units (*e.g.*, rad/s, dBΩ, and degrees).

$$Z_{in}(j\omega) = \frac{R_1}{1 + j\omega R_1 C} \quad : \quad 1 \text{ pole @ } s = j\omega = -\frac{1}{R_1 C} = -10^3 \ \frac{\text{rad}}{\text{s}}$$



3. **[45 pts]** Consider the stress transducer below, with constant supply voltage $V_s = 3$ V, two identical current sources $I_{\text{ref}} = 10\ \mu$A, a resistor R, and a strain gauge R_G with nominal resistance $R_{\text{nom}} = 100$ kΩ, gauge factor $G = 100$, and Young's modulus $E = 100$ kPa. The transducer produces a differential output voltage V_o in response to a stress σ applied to the strain gauge.



$$
\begin{aligned}
R_G &= R_{\text{nom}}\,(1 + G\,\epsilon) \\
\sigma &= E\,\epsilon
\end{aligned}
$$

(a) [10 pts] Find the output voltage V_o as a function of the applied stress σ. Does it depend on the supply voltage, and why?

$$
\begin{aligned}
V_o &= R_G \cdot I_{ref} - R \cdot I_{ref} \\
&= R_{nom}\left(1 + \frac{G}{E}\sigma\right) I_{ref} - R \cdot I_{ref}
\end{aligned}
$$

INDEPENDENT of supply voltage V_s
for ideal current sources I_{ref}

$$V_O = \underbrace{(R_{nom} - R) \cdot I_{ref}}_{\substack{V_{off} \\ \text{OFFSET} \\ 3(b)}} + \underbrace{R_{nom} \cdot I_{ref} \cdot \frac{G}{E}}_{\substack{S \\ \text{SENSITIVITY} \\ 3(c)}} \cdot \sigma$$

(b) [5 pts] Find the value of resistance R that nulls the offset of the transducer. Use this value of R for the remainder of this problem.

3(a) : $V_{off} = 0$ for $R = R_{nom} = 100\ k\Omega$

(c) [5 pts] Find the sensitivity of the transducer.

3(a) : $S = R_{nom} \cdot I_{ref} \cdot \frac{G}{E} = 100\,k\Omega \cdot 10\,\mu A \cdot \frac{100}{100\,kPa}$

$= 1\ V/kPa$

(d) [10 pts] Due to mismatch in manufacturing of the instrument, the left current source I_{ref} is actually 100 nA lower than expected, whereas the right current source I_{ref} is actually 100 nA higher than expected. Find the absolute accuracy of stress measurement by the instrument at $\sigma = 0$, and at $\sigma = 1$ kPa.

$$V_{O\,actual} = R_{nom}\left(1 + \frac{G}{E}\sigma\right)\left(I_{ref} + 100nA\right) - R_{nom}\left(I_{ref} - 100nA\right)$$

$$V_{O\,ideal} = R_{nom}\left(1 + \frac{G}{E}\sigma\right) I_{ref} - R_{nom} I_{ref}$$

$$\text{Accuracy}(\sigma) = \frac{\text{Accuracy}(V_O)}{S}$$

$$= \frac{1}{S} \cdot \left| V_{O\,actual} - V_{O\,ideal} \right|$$

$$= \frac{1}{S} \cdot \left| R_{nom}\left(1 + \frac{G}{E}\sigma\right) \cdot 100nA - R_{nom} \cdot (-100nA) \right|$$

$$= \frac{R_{nom} \cdot 100nA}{S} \cdot \left| 2 + \frac{G}{E}\sigma \right|$$

$$= 10\text{ Pa} \cdot \left| 2 + \frac{\sigma}{1\text{ kPa}} \right|$$

$\sigma = 0 \Rightarrow \text{Accuracy}(\sigma) = 10\text{ Pa} \cdot 2 = 20\text{ Pa}$

$\sigma = 1\text{ kPa} \Rightarrow \text{Accuracy}(\sigma) = 10\text{ Pa} \cdot \left(2 + \frac{1\text{ kPa}}{1\text{ kPa}}\right) = 30\text{ Pa}$

(e) [10 pts] An analog-to-digital converter (ADC) is used to digitize the voltage output V_o for a digital reading of the stress σ. Find the full-scale voltage range and the number of bits of the ADC required for a full-scale stress range from 0 to 1 kPa at a resolution of 1 Pa.

$$\sigma = 0 \quad \Rightarrow \quad V_o = V_{off} = 0$$

$$\sigma = 1\ \text{kPa} \quad \Rightarrow \quad V_o = V_{off} + \frac{1\ \text{kPa}}{1\ \text{kPa/V}} = 1\ \text{V}$$

$\Rightarrow$ ADC voltage range : 0 to 1 V

$$\Delta\sigma = 1\ \text{Pa} \quad \Rightarrow \quad \Delta V_o = \frac{1\ \text{Pa}}{1\ \text{kPa/V}} = 1\ \text{mV}$$

$$\Rightarrow \text{ADC number of bits} = \log_2 \underbrace{\frac{1\ \text{V}}{1\ \text{mV}}}_{1{,}000} = 10\ \text{bits}$$

(f) [5 pts] You observe that the transducer is subject to a drifting offset due to variations in temperature. What simple change could you make to the design in order to eliminate this temperature dependence? *Hint*: you may assume that all identical components, such as the two current sources, have identical temperature coefficients.

Replace R with a matching strain gage R_G, but with zero stress: