BENG 186B Winter 2023

Quiz 1

Friday, January 27, 2023

Name *(Last, First):* SOLUTIONS

- This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due January 27, 2023 at 11:59pm, over Canvas (Gradescope). It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[20 pts]** Circle the **best answer (only one answer per question)**:

   (a) [4 pts] The precision of a bioinstrument is affected by:

      i. systematic offset.
      ii. measurement noise.
      iii. saturation.
      iv. all of the above.

   (b) [4 pts] An inductor-capacitor undamped second-order system:

      i. oscillates at its natural frequency.
      ii. produces unbounded output when driven with an input at the natural frequency.
      iii. conserves energy.
      iv. all of the above.

   (c) [4 pts] The Norton equivalent of a linear circuit is:

      i. an ideal voltage source in parallel with an impedance.
      ii. an ideal voltage source in series with an impedance.
      iii. an ideal current source in series with an impedance.
      iv. an ideal current source in parallel with an impedance.

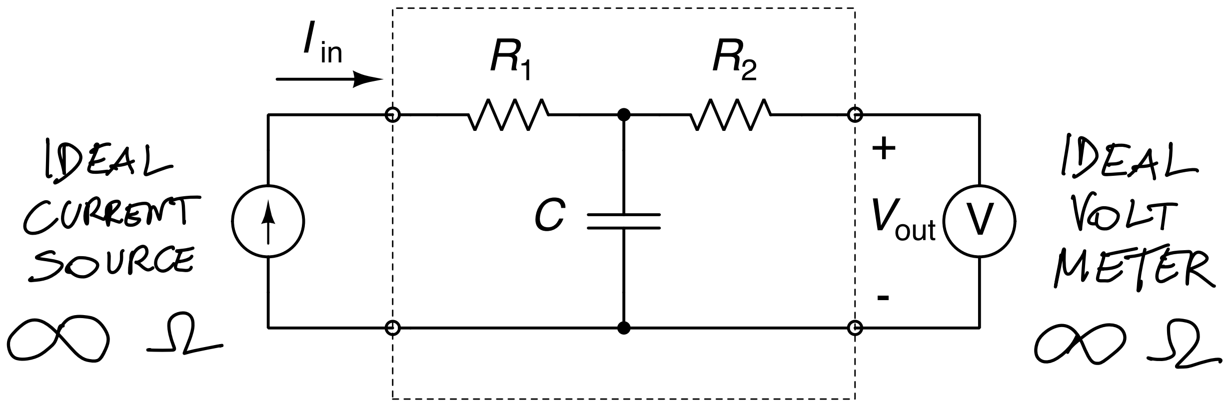
   (d) [4 pts] The gauge factor of a strain gauge depends on:

      i. resistance.
      ii. Young's modulus.
      iii. Poisson ratio.
      iv. all of the above.

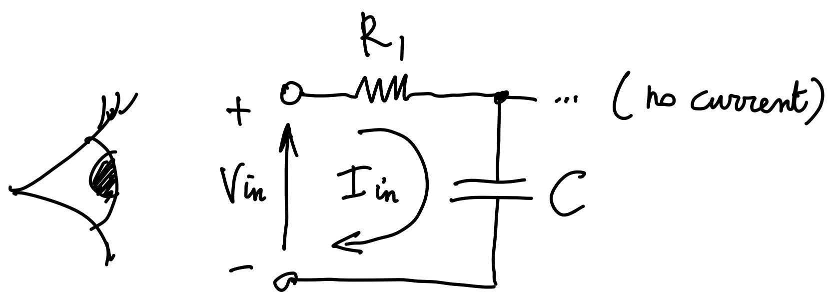
   (e) [4 pts] The sensitivity of an inductive displacement sensor depends on:

      i. the number of windings of the inductive coil.
      ii. the magnetic susceptibility of the magnetic core.
      iii. its geometry.
      iv. all of the above.

2. **[40 pts]** Consider the **current-input, voltage-output** filter circuit below.

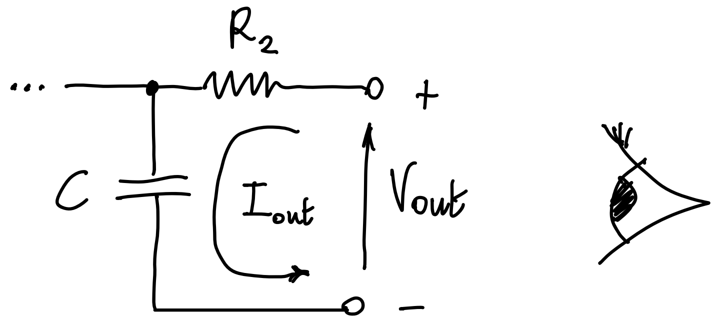


(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$.



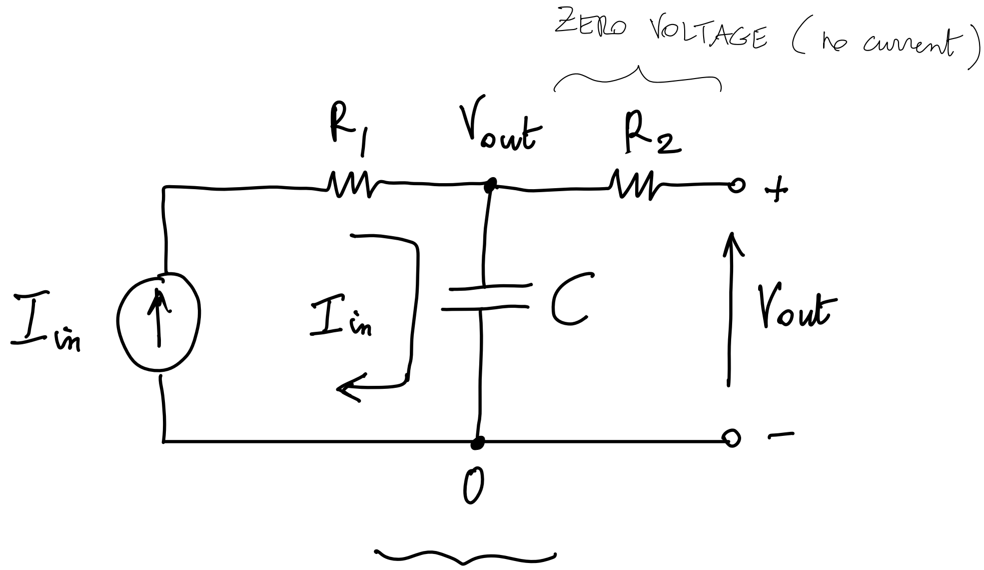
$$Z_{in} = R_1 + \frac{1}{j\omega C} = \frac{1 + j\omega R_1 C}{j\omega C}$$

(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.



$$Z_{out} = R_2 + \frac{1}{j\omega C} = \frac{1 + j\omega R_2 C}{j\omega C}$$

(c) [10 pts] Find the transfer function $H(j\omega) = V_{out}(j\omega) \,/\, I_{in}(j\omega)$. What are the units?

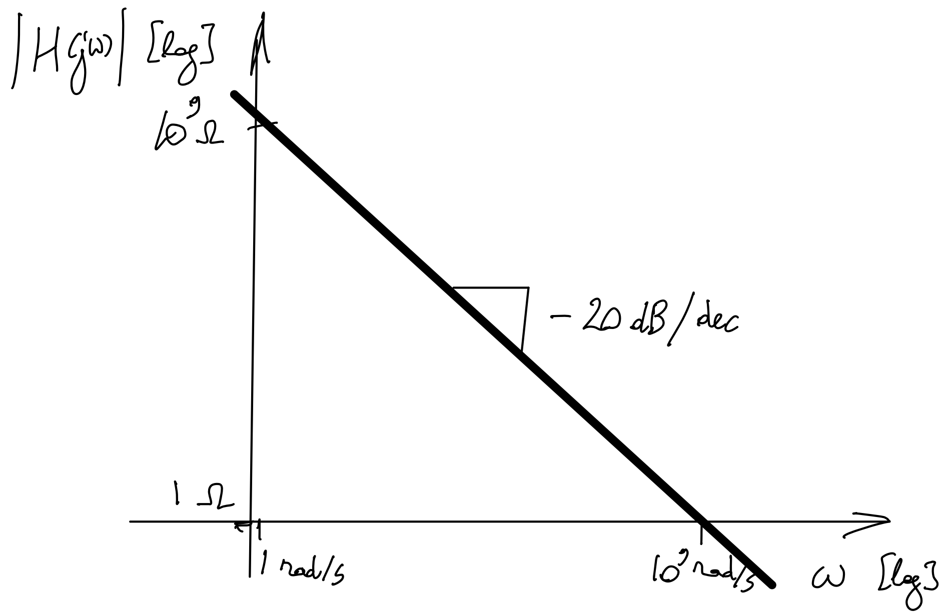


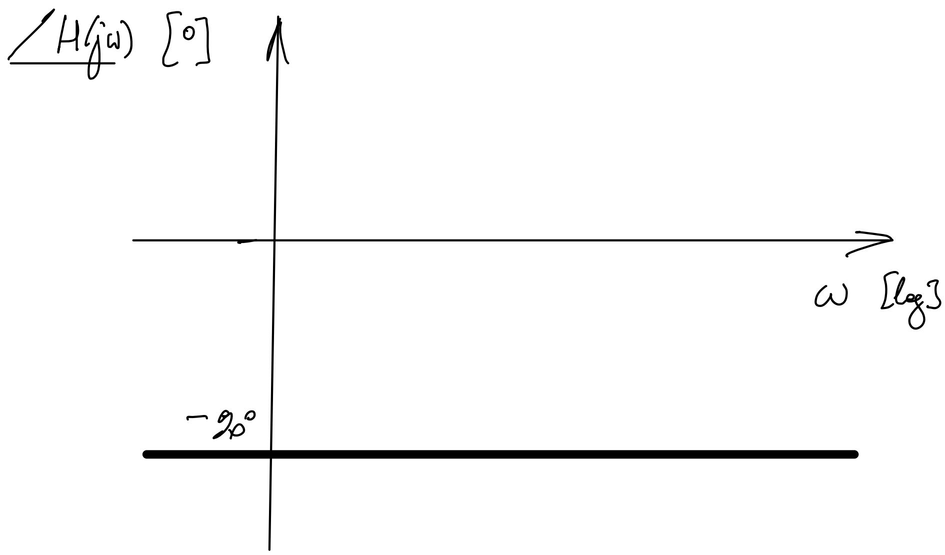
$$V_{out} = \frac{1}{j\omega C} I_{in}$$

$$\Rightarrow H(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)} = \frac{1}{j\omega C}$$

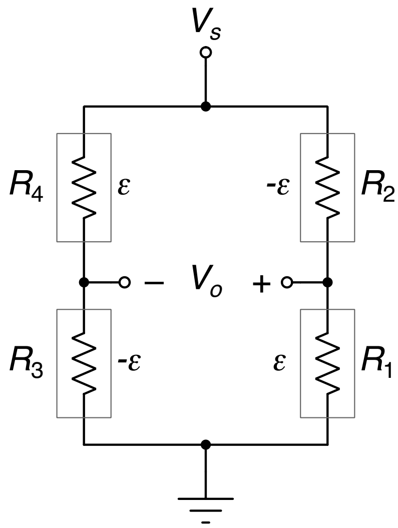
(d) [10 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C = 1$ nF, and $R_1 = R_2 = 1$ kΩ. Be sure to label the axes and indicate the units.

$$H(j\omega) = \frac{1}{j\omega C} \quad : \quad \begin{cases} \text{NO ZEROS} \\ \text{POLE @ } s = 0 \end{cases}$$





3. **[40 pts]** Consider the strain transducer below, with constant supply voltage $V_s = 1$ V, and four strain gauges R_1, R_2, R_3 and R_4 all with identical nominal resistance $R_{nom} = 100$ kΩ, and gauge factor $G = 100$. The transducer produces a differential output voltage V_o in response to positive strain ϵ applied to two of the strain gauges R_1 and R_4, and exact opposite negative strain $-\epsilon$ applied to the other two strain gauges R_2 and R_3.



$$
\begin{aligned}
R_1 &= R_4 = R_{nom}\,(1 + G\,\epsilon) \\
R_2 &= R_3 = R_{nom}\,(1 - G\,\epsilon)
\end{aligned}
$$

(a) [5 pts] Find the output voltage V_o as a function of strain ϵ. Is the response linear, and why?

$$V_o = \left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) V_s$$

$$= \left(\frac{\cancel{R_{nom}}(\cancel{1}+G\varepsilon)}{2\cancel{R_{nom}}} - \frac{\cancel{R_{nom}}(\cancel{1}-G\varepsilon)}{2\cancel{R_{nom}}} \right) V_s$$

$$= \frac{\cancel{2}G\varepsilon}{\cancel{2}} V_s = G V_s \cdot \varepsilon \quad \text{LINEAR}$$

because differential with complementary strain gauges

(b) [5 pts] Find the sensitivity and offset of the strain transducer.

Sensitivity $S = \frac{dV_o}{d\varepsilon} = GV_s = 100\ V$

Zero offset

(c) [10 pts] Now consider that the gauge factors of both R_1 and R_2 are 10 % smaller than expected, whereas the gauge factors of both R_3 and R_4 are 10 % larger than expected. Find the sensitivity and offset of the strain transducer, and compare with (b). Explain what you observe.

$$V_o = \left(\frac{R_{nom}(\cancel{1} + 0.9G\varepsilon)}{R_{nom}(1 + \cancel{0.9G\varepsilon} + 1 \cancel{- 0.9G\varepsilon})} - \frac{R_{nom}(\cancel{1} - 1.1G\varepsilon)}{R_{nom}(1 \cancel{- 1.1G\varepsilon} + 1 \cancel{+ 1.1G\varepsilon})} \right) V_s$$

$$= \frac{\cancel{R_{nom}}\, \cancel{2} G\varepsilon}{\cancel{R_{nom}}\, \cancel{2}} V_s = G V_s \varepsilon$$

Sensitivity $S = \frac{dV_o}{d\varepsilon} = G V_s = 100\ V$

Zero offset

Same as (b) !

Because, the variations cancel in the differential complementary combining top and bottom.

(d) [10 pts] Now consider that the strain gauges R_1 and R_3 have identical temperature coefficients $\alpha = 0.01\ \mathrm{C}^{-1}$, whereas the strain gauges R_2 and R_4 are temperature independent. Find the sensitivity of the transducer output voltage V_o to temperature T, at zero strain $\epsilon = 0$. Explain what you observe.

$$V_o \underset{@\ \varepsilon = 0}{=} \left(\frac{R_{nom}(1+\alpha\Delta T)}{R_{nom}(1+\alpha\Delta T) + R_{nom}} - \frac{R_{nom}(1+\alpha\Delta T)}{R_{nom}(1+\alpha\Delta T) + R_{nom}} \right) V_s$$

$$= 0$$

Zero sensitivity to temperature!

Because, differential with identical variations left and right.

(e) [10 pts] Find the worst-case absolute accuracy of digital reading of the strain using a 12-bit analog-to-digital converter that spans a 100 mV range at the output of the strain transducer.

Worst-case absolute accuracy (strain)

$$= \frac{1}{2} \text{ LSB @ input} \quad \text{(strain)}$$

$$= \frac{1}{2} \frac{\text{LSB @ output}}{S} \quad \text{(voltage)}$$

$$\left(\text{Sensitivity} = \frac{\text{output}}{\text{input}} \right)$$

$$= \frac{1}{2} \frac{2^{-12} \text{ Range @ output}}{S}$$

$$= \frac{1}{2} \frac{\frac{1}{4{,}096} \, 0.1\,V}{100\,V} = 1.22 \; 10^{-7}$$

(12.2 ppm)