BENG 186B Winter 2023

Quiz 1

Friday, January 27, 2023

Name (Last, First): SOLUTIONS

• This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.

• The quiz is due January 27, 2023 at 11:59pm, over Canvas (Gradescope). It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.

• There are 3 problems. Points for each problem are given in [brackets]. There are 100 points total.
1. [20 pts] Circle the **best answer (only one answer per question)**:

(a) [4 pts] The precision of a bioinstrument is affected by:
   - i. systematic offset.  
   - ii. measurement noise. 
   - iii. saturation.  
   - iv. all of the above.

(b) [4 pts] An inductor-capacitor undamped second-order system:
   - i. oscillates at its natural frequency. 
   - ii. produces unbounded output when driven with an input at the natural frequency. 
   - iii. conserves energy. 
   - iv. all of the above.

(c) [4 pts] The Norton equivalent of a linear circuit is:
   - i. an ideal voltage source in parallel with an impedance. 
   - ii. an ideal voltage source in series with an impedance. 
   - iii. an ideal current source in series with an impedance. 
   - iv. an ideal current source in parallel with an impedance.

(d) [4 pts] The gauge factor of a strain gauge depends on:
   - i. resistance. 
   - ii. Young’s modulus. 
   - iii. Poisson ratio. 
   - iv. all of the above.

(e) [4 pts] The sensitivity of an inductive displacement sensor depends on:
   - i. the number of windings of the inductive coil. 
   - ii. the magnetic susceptibility of the magnetic core. 
   - iii. its geometry. 
   - iv. all of the above.
2. **[40 pts]** Consider the **current-input, voltage-output** filter circuit below.

(a) **[10 pts]** Find the input impedance $Z_{in}(j\omega)$.

\[
Z_{in} = R_1 + \frac{1}{j\omega C} = \frac{1 + j\omega R_1 C}{j\omega C}
\]
(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.

$$Z_{out} = R_2 + \frac{1}{j\omega C} = \frac{1 + j\omega R_2 C}{j\omega C}$$
(c) [10 pts] Find the transfer function $H(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$. What are the units?

\[
\mathbf{H}(j\omega) = \frac{\mathbf{V}_{out}(j\omega)}{\mathbf{I}_{in}(j\omega)} = \frac{1}{j\omega \mathbf{C}}
\]
(d) [10 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C = 1 \text{ nF}$, and $R_1 = R_2 = 1 \text{ k}\Omega$. Be sure to label the axes and indicate the units.

$$H(j\omega) = \frac{1}{j\omega C} : \begin{cases} \text{NO ZEROS} \\ \text{POLE @ } s = 0 \end{cases}$$
3. [40 pts] Consider the strain transducer below, with constant supply voltage $V_s = 1$ V, and four strain gauges $R_1$, $R_2$, $R_3$ and $R_4$ all with identical nominal resistance $R_{nom} = 100$ kΩ, and gauge factor $G = 100$. The transducer produces a differential output voltage $V_o$ in response to positive strain $\epsilon$ applied to two of the strain gauges $R_1$ and $R_4$, and exact opposite negative strain $-\epsilon$ applied to the other two strain gauges $R_2$ and $R_3$.

![Strain Transducer Diagram]

$$
R_1 = R_4 = R_{nom} (1 + G \epsilon)
$$
$$
R_2 = R_3 = R_{nom} (1 - G \epsilon)
$$

(a) [5 pts] Find the output voltage $V_o$ as a function of strain $\epsilon$. Is the response linear, and why?

$$
V_o = \left( \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) V_s
$$

$$
= \left( \frac{R_{nom} (1+G\epsilon)}{2 R_{nom}} - \frac{R_{nom} (1-G\epsilon)}{2 R_{nom}} \right) V_s
$$

$$
= \frac{G \epsilon}{2} V_s = GV_s \cdot \epsilon \quad \text{LINEAR}
$$

because differential with complementary strain gauges

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**Solution:**

The output voltage $V_o$ can be derived from the given circuit configuration. By applying the voltage divider rule to the circuit, we can express $V_o$ as a function of $V_s$ and the strain $\epsilon$. The strain gauges have a differential response due to the opposite strain applied to $R_1$ and $R_4$, and $R_2$ and $R_3$. The gauge factor $G$ amplifies the strain effect, making the output voltage linear with respect to strain.
(b) [5 pts] Find the sensitivity and offset of the strain transducer.

\[
S = \frac{dV_o}{d\varepsilon} = G V_s = 100 \text{ V}
\]
(c) [10 pts] Now consider that the gauge factors of both $R_1$ and $R_2$ are 10% smaller than expected, whereas the gauge factors of both $R_3$ and $R_4$ are 10% larger than expected. Find the sensitivity and offset of the strain transducer, and compare with (b). Explain what you observe.

\[
V_o = \left( \frac{R_{\text{nom}}(Y + 0.5GE)}{R_{\text{nom}}(1 + 0.5GE)} - \frac{R_{\text{nom}}(Y - 1.1GE)}{R_{\text{nom}}(1 - 1.1GE)} \right) V_S
\]

\[
= \frac{R_{\text{nom}} \times GE}{R_{\text{nom}} \times Y} V_S = G V_S \varepsilon
\]

Sensitivity \quad S = \frac{dV_o}{d\varepsilon} = GV_S = 100 \text{ V}

Zero offset

Same as (b)!

Because, the variations cancel in the differential complementary combining top and bottom.
(d) [10 pts] Now consider that the strain gauges $R_1$ and $R_3$ have identical temperature coefficients $\alpha = 0.01 \, \text{C}^{-1}$, whereas the strain gauges $R_2$ and $R_4$ are temperature independent. Find the sensitivity of the transducer output voltage $V_o$ to temperature $T$, at zero strain $\epsilon = 0$. Explain what you observe.

$$
V_o = \left( \frac{R_{\text{nom}} (1 + \alpha \Delta T)}{R_{\text{nom}} (1 + \alpha \Delta T) + R_{\text{nom}}} - \frac{R_{\text{nom}} (1 + \alpha \Delta T)}{R_{\text{nom}} (1 + \alpha \Delta T) + R_{\text{nom}}} \right) V_s
$$

$$
\text{at } \epsilon = 0
= 0
$$

Zero sensitivity to temperature!

Because, differential with identical variations left and right.
(e) [10 pts] Find the worst-case absolute accuracy of digital reading of the strain using a 12-bit analog-to-digital converter that spans a 100 mV range at the output of the strain transducer.

\[
\text{Worst-case absolute accuracy (strain)} = \frac{1}{2^{12}} \text{LSB @ output (volts)} = \frac{1}{2^{12}} \times 0.1 V = 1.22 \times 10^{-7} \text{ (12.2 ppm)}
\]