BENG 186B Winter 2025

Quiz 1

Friday, January 24, 2025

Name *(Last, First):* SOLUTIONS

- This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due January 24, 2025 at 11:59pm, over Canvas (Gradescope). It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[20 pts]** Circle the **best answer (only one answer per question)**:

   (a) [4 pts] The accuracy of a bioinstrument can be improved by:

      i. averaging multiple output samples.
      ii. lowering input noise.
      iii. subtracting output offset.
      iv. all of the above.

      } these improve precision

   (b) [4 pts] A resistor-capacitor first-order system:

      i. peaks at its natural frequency.
      ii. produces exponentially decaying output transients.
      iii. conserves energy.
      iv. none of the above.

   (c) [4 pts] The Thévenin equivalent of a linear circuit is:

      i. an ideal voltage source in parallel with an impedance.
      ii. an ideal voltage source in series with an impedance.
      iii. an ideal current source in series with an impedance.
      iv. an ideal current source in parallel with an impedance.

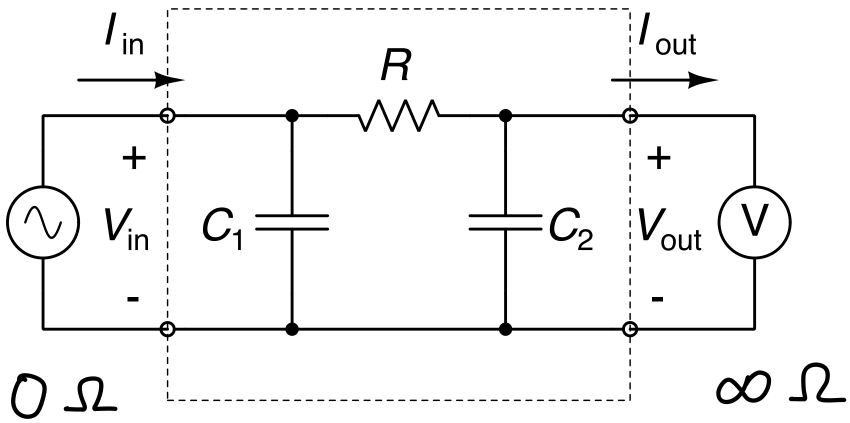
   (d) [4 pts] The resistance of a strain gauge depends on:

      i. strain.
      ii. stress.
      iii. temperature.
      iv. all of the above.

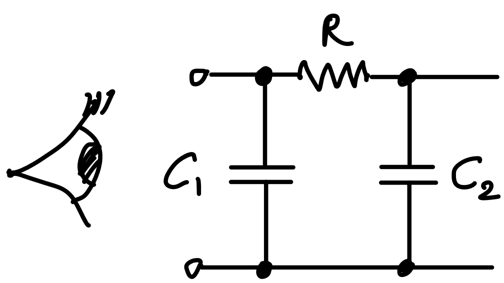
   (e) [4 pts] An ideal transformer:

      i. perfectly couples magnetic field across two coils.
      ii. transfers power without loss.
      iii. insulates input from output.
      iv. all of the above.

2. **[40 pts]** Consider the **voltage-input, voltage-output** filter circuit below.



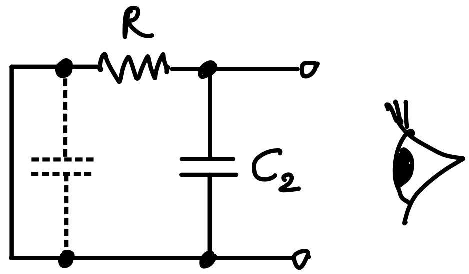
(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$.



$$Z_{in} = \frac{1}{j\omega C_1} \parallel R + \frac{1}{j\omega C_2} = \frac{\frac{1}{j\omega C_1}\left(R + \frac{1}{j\omega C_2}\right)}{\frac{1}{j\omega C_1} + R + \frac{1}{j\omega C_2}} \quad \begin{matrix} \times (j\omega C_1)(j\omega C_2) \\ \times (j\omega C_1)(j\omega C_2) \end{matrix}$$

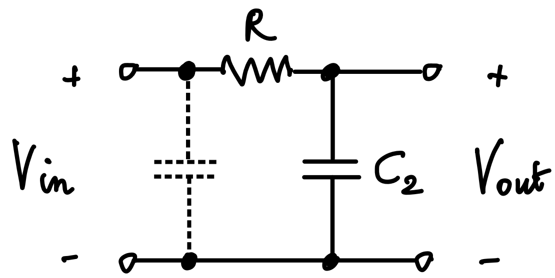
$$= \frac{1 + j\omega R C_2}{j\omega\left(C_1 + C_2 + j\omega R C_1 C_2\right)}$$

(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.



$$Z_{out} = \frac{1}{j\omega C_2} \parallel R = \frac{\frac{1}{j\omega C_2} \cdot R}{\frac{1}{j\omega C_2} + R} = \frac{R}{1 + j\omega R C_2}$$

(c) [10 pts] Find the transfer function $H(j\omega) = V_{out}(j\omega) / V_{in}(j\omega)$. What are the units?

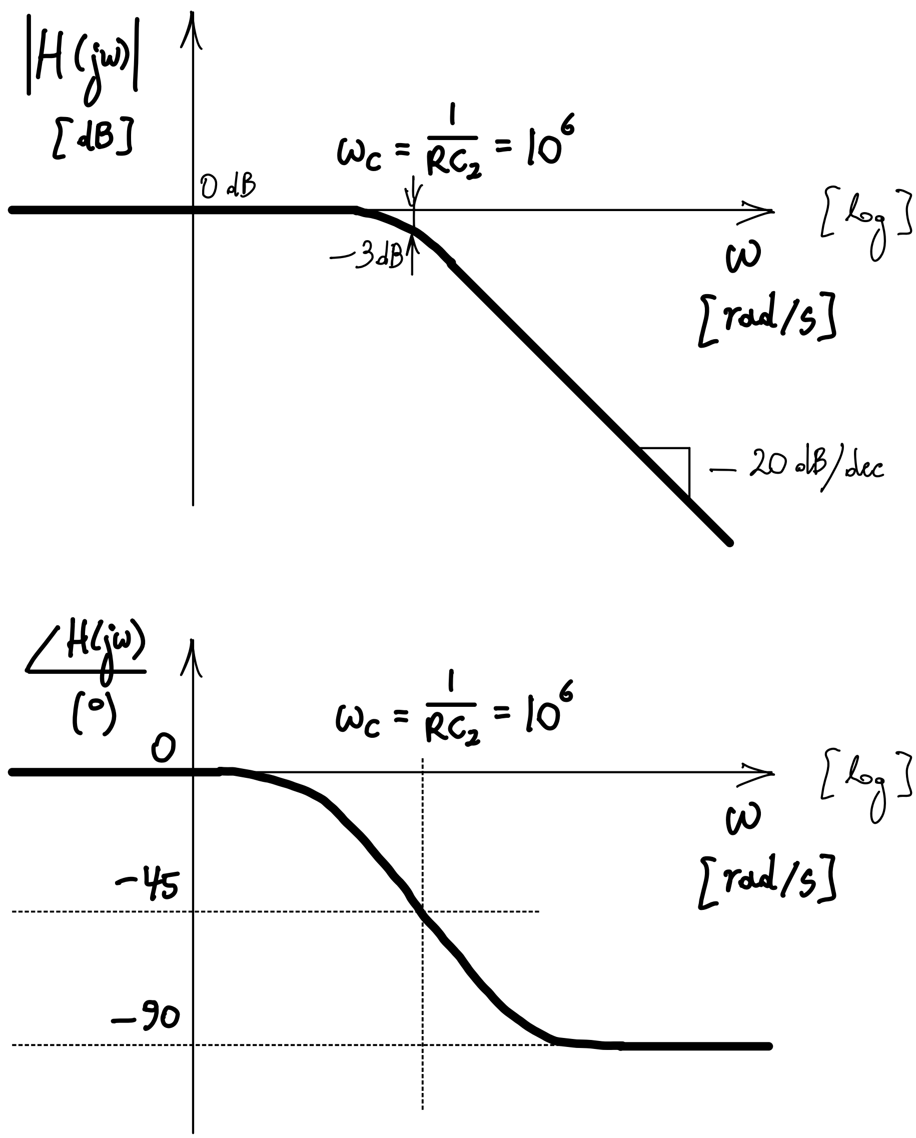


$$V_{out} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2} + R} V_{in} = \frac{1}{1 + j\omega RC_2} V_{in}$$

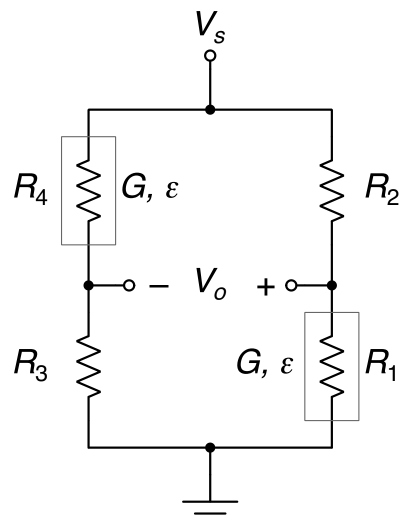
$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + j\omega RC_2}$$

Units: dimensionless (V / V)

(d) [10 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C_1 = C_2 = 1$ nF, and $R = 1$ kΩ. Be sure to label the axes and indicate the units.



3. **[40 pts]** Consider the stress transducer below, with constant supply voltage $V_s = 3$ V, and four resistors R_1, R_2, R_3 and R_4 all with identical nominal resistance $R_{nom} = 10$ kΩ. Resistors R_1 and R_4 are strain gauges with identical gauge factor $G = 50$, and identical Young's modulus $E = 10$ kPa. The transducer produces a differential output voltage V_o in response to stress σ applied to the two strain gauges R_1 and R_4.



$$
\begin{aligned}
R_1 &= R_4 = R_{nom}\,(1 + G\,\epsilon) \\
R_2 &= R_3 = R_{nom}
\end{aligned}
$$

(a) [10 pts] Find the output voltage V_o as a function of stress σ. Is the response linear, and why?

$$V_o = \left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) V_s$$

$$= \left(\frac{R_{nom}\left(1 + \frac{G}{E}\sigma\right)}{R_{nom}\left(1 + \frac{G}{E}\sigma\right) + R_{nom}} - \frac{R_{nom}}{R_{nom} + R_{nom}\left(1 + \frac{G}{E}\sigma\right)} \right) V_s$$

$$= \frac{\cancel{R_{nom}}\,\frac{G}{E}\sigma}{\cancel{R_{nom}}\left(2 + \frac{G}{E}\sigma\right)} V_s = \frac{\frac{1}{2}\frac{G}{E} V_s\,\sigma}{1 + \frac{1}{2}\frac{G}{E}\sigma} \quad \leftarrow \text{Nonlinear}$$

(b) [5 pts] Find the offset of the stress transducer.

$V_{off} = 0$ because $V_o(\sigma = 0) = 0$

(c) [5 pts] Find the sensitivity of the stress transducer at zero stress.

$$S = \left.\frac{dV_o}{d\sigma}\right|_{\sigma=0} = \frac{1}{2}\frac{G}{E}V_s = \frac{1}{2}\frac{50}{10\,kPa}3V = 7.5\,\frac{V}{kPa}$$

(d) [10 pts] Now consider that the resistors R_2 and R_3 have identical temperature coefficient α, whereas the strain gauges R_1 and R_4 are temperature independent. Find the sensitivity of the transducer output voltage V_o to temperature T, at zero stress $\sigma = 0$. Explain what you observe.

$$R_1 = R_4 = R_{nom} \quad @ \; \sigma = 0$$

$$R_2 = R_3 = R_{nom}(1 + \alpha \Delta T)$$

$$V_o = \left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) V_s$$

$$= \left(\frac{R_{nom}}{R_{nom} + R_{nom}(1 + \alpha\Delta T)} - \frac{R_{nom}(1 + \alpha \Delta T)}{R_{nom}(1 + \alpha\Delta T) + R_{nom}} \right) V_s$$

$$= - \frac{\cancel{R_{nom}}\, \alpha \Delta T}{\cancel{R_{nom}}(2 + \alpha \Delta T)} V_s = - \frac{\frac{\alpha}{2} V_s \Delta T}{1 + \frac{\alpha}{2} \Delta T}$$

$$S_T = \left. \frac{\partial V_o}{\partial \Delta T} \right|_{\sigma = 0} = -\frac{\alpha}{2} V_s \quad \text{for } |\Delta T| \ll \frac{2}{\alpha}$$

The offset of the stress transducer shifts with temperature.

Regardless of stress it also acts as a temperature transducer.

(e) [10 pts] Find the worst-case absolute accuracy of digital reading of the stress using a 10-bit analog-to-digital converter that spans a 200 mV range at the output of the strain transducer.

Worst-case absolute accuracy

$$= \frac{1}{2} \text{ LSB @ input} \qquad \text{(stress)}$$

$$= \frac{1}{2} \frac{\text{LSB @ output}}{S} \qquad \text{(voltage)}$$

$$= \frac{1}{2} \frac{2^{-10} \text{ Range @ output}}{S}$$

$$= \frac{1}{2} \frac{\frac{1}{1{,}024} \; 0.2 \, \cancel{V}}{7.5 \frac{\cancel{V}}{kPa}}$$

$$= 1.3 \, 10^{-5} \text{ kPa} = 0.013 \text{ Pa}$$