BENG 186B Winter 2021

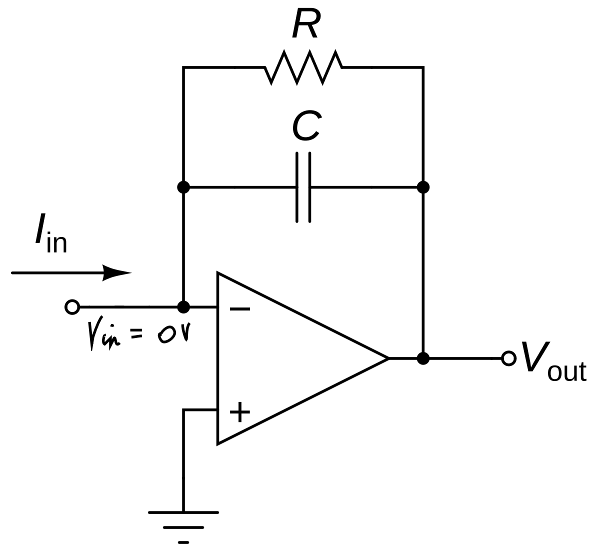
Quiz 2

Friday, February 12, 2021

Last Name, First Name: SOLUTIONS

- This quiz is on-line, open-book, and open-notes. You may use a calculator or an equivalent program, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due February 12, 2021 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[25 pts]** Consider the current-in, voltage-out active filter circuit below:



(a) [15 pts] Assume the operational amplifier is ideal and unsaturated. Derive the transfer function $H(j\omega) = V_{out}(j\omega) \,/\, I_{in}(j\omega)$. What type of filter is this? What is the cutoff frequency?

Ohm's law : $0 - V_{out} = \left(R \,\|\, \frac{1}{j\omega C}\right) \cdot I_{in}$

$$\Rightarrow \quad H(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)} = -\left(R \,\|\, \frac{1}{j\omega C}\right)$$

$$= -\frac{R}{1 + j\omega RC}$$

Low-pass filter with cut-off $\omega_c = \frac{1}{RC}$

$$f_c = \frac{1}{2\pi RC}$$

(b) [5 pts] What is the input impedance at the I_{in} node?

$V_{in} = 0$ (virtual ground).

$$\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = 0$$

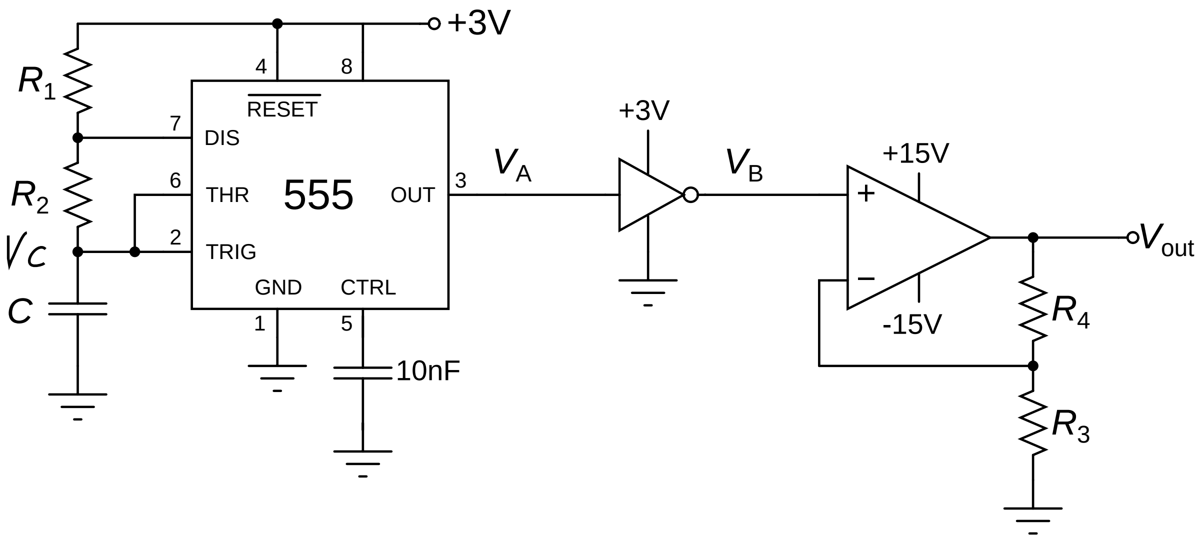
(c) [5 pts] What is the output impedance at the V_{out} node?

V_{out} does not depend on I_{out}.

$$\Rightarrow Z_{out} = \frac{\Delta V_{out}}{\Delta I_{out}} = 0$$

2. **[35 pts]** Consider the signal generator circuit shown below. All active components are ideal. The 555 timer IC and the inverter logic gate operate from a +3V single supply, while the opamp operates from a +15V/−15V dual supply. The values for the passive components are $R_1 = 1.143\,\text{M}\Omega$, $R_2 = 143\,\text{k}\Omega$, $R_3 = 50\,\text{k}\Omega$, $R_4 = 150\,\text{k}\Omega$, and $C = 100\,\text{nF}$. You may also find the following equations useful for the 555 timer ($\ln(3) \approx 1.1$ and $\ln(2) \approx 0.7$):

$$T = \ln(3) \times RC \qquad T_{lo} = \ln(2) \times R_2 C \qquad T_{hi} = \ln(2) \times (R_1 + R_2)C$$



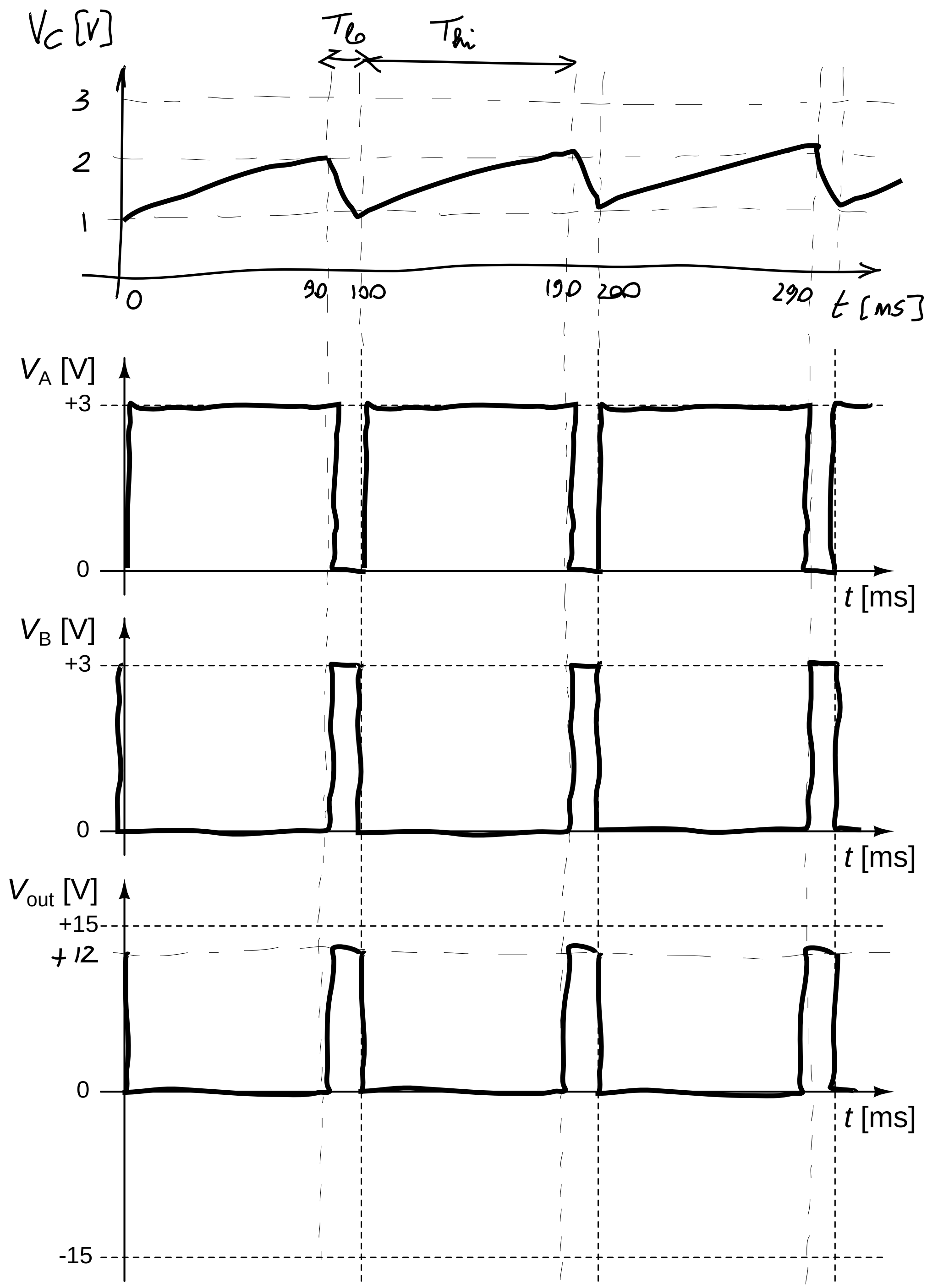
(a) [15 pts] Sketch the waveforms for the voltages V_A, V_B and V_{out} on the diagrams on the next page. You may assume that at time $t = 0$ the voltage on the capacitor C is 1 V.

V_A : 555 timer: astable (oscillator) with: $T_{lo} = 0.7\, R_2 C = 10\text{ ms}$

$T_{hi} = 0{,}7\,(R_1 + R_2)C = 90\text{ ms}$

$V_A \rightarrow V_B$: Inverter: logical inversion

$V_B \rightarrow V_{out}$: Non-inverting amplifier with gain: $A_v = 1 + \dfrac{R_4}{R_3} = 4$

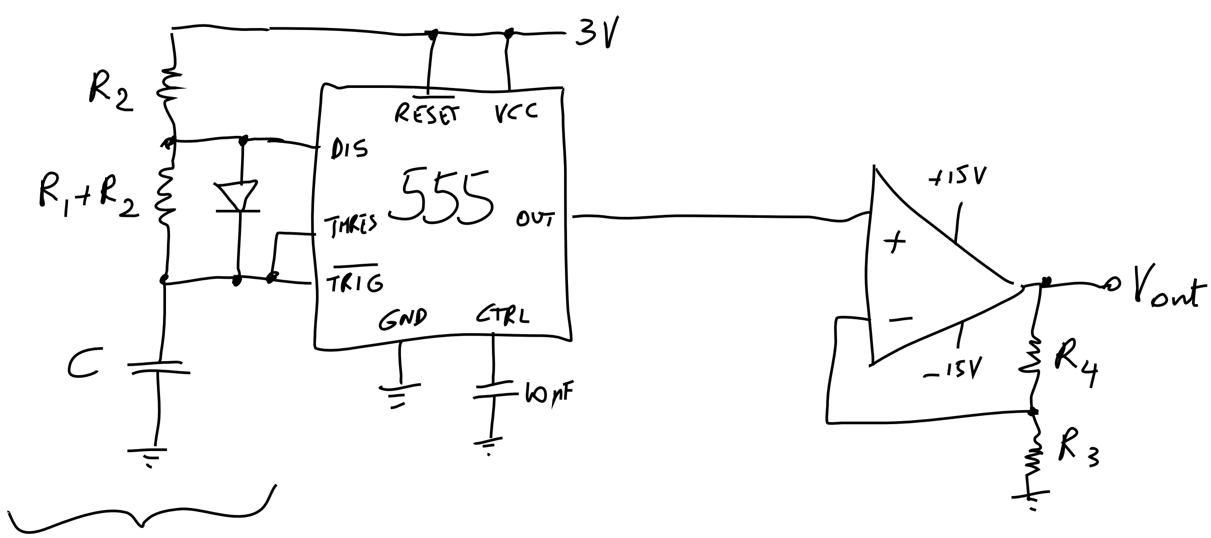


V_C [V]
T_{lo}
T_{hi}
3
2
1
0
90
100
190
200
290
t [ms]
V_A [V]
+3
0
t [ms]
V_B [V]
+3
0
t [ms]
V_{out} [V]
+15
+12
0
t [ms]
-15

(b) [15 pts] What purpose does the inverter serve? Can you come up with a variation on this circuit, without the inverter or any other logic gates, that generates the same voltage waveform for V_{out}?

The 555 astable (oscillator) output has a duty cycle always greater than 0.5.
The inverter inverts its polarity, so that the duty cycle is always less than 0.5.

Many variations are possible to arrive at the same output waveform, e.g.:



$T_{lo} = 0.7(R_1+R_2)C$ (diode is an open circuit during discharge)

$T_{hi} = 0.7\,R_2 C$ (diode is a short circuit during charge)

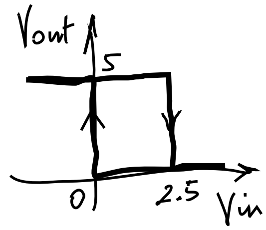
(c) [5 pts] How does the output voltage V_{out} change if the -15V lower supply of the opamp is replaced with ground (0V), and its +15V upper supply is replaced with +9V?

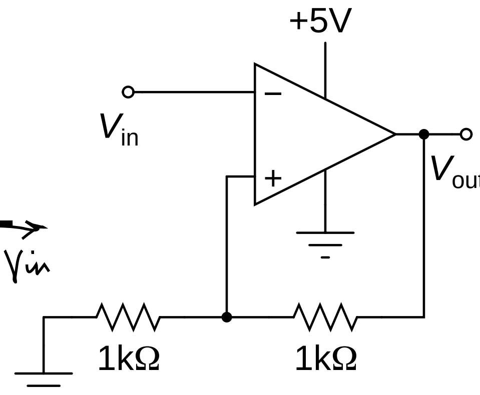
The opamp clips the output to 0V from below, and to +9V from above.

This scales the output by 75%.

3. **[20 pts]** Circle the **best answer (only one answer per question)**:

(a) [4 pts] The output V_{out} of the circuit at right (with an ideal opamp) goes high when the input V_{in} goes:





i. below 0

ii. above 0

iii. below +2.5V

iv. above +2.5V

v. above +5V

(b) [4 pts] The virtual ground in an active circuit is established:

i. by shorting the input pins of the opamp

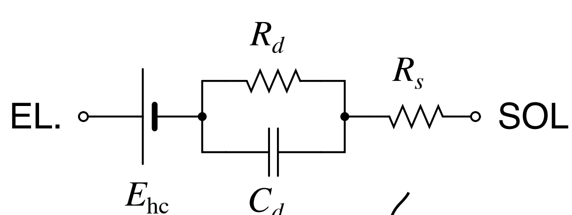
ii. by saturating the output of the opamp

iii. by grounding the opamp inverting input

iv. through high-gain negative feedback

v. through high-gain positive feedback

(c) [4 pts] At very high frequencies the electrode shown on the right has:



i. infinite impedance

ii. low impedance $= R_s$

iii. zero impedance

iv. zero voltage

v. voltage near the half-cell potential

$\left(R_s \ll R_d \parallel \frac{1}{j\omega C_d} \right)$

(d) [1 pt ea.] Indicate for each statement below whether it is true or false:

i. **TRUE** / **FALSE**: Nernst potentials result from the thermal equilibrium between diffusion and drift of a single type of ions permeating through the membrane.

ii. **TRUE** / **FALSE**: ECoG can be measured directly using electrodes placed on the scalp.

iii. **TRUE** / **FALSE**: ENG activity is larger in amplitude than EMG activity.

iv. **TRUE** / **FALSE**: EOG measures the response of the retina in the eye to a flash of light.

v. **TRUE** / **FALSE**: The biopotential generated by a current dipole is zero anywhere in the plane orthogonal to the dipole through its center.

vi. **TRUE** / **FALSE**: The Q wave of ECG indicates atrial depolarization.

vii. **TRUE** / **FALSE**: The battery voltage of an electrochemical cell is given by the difference between the electrode half-cell potentials.

viii. **TRUE** / **FALSE**: A non-contact electrode is non-polarizable.

4. **[20 pts]** Consider an intracellular measurement of the action potential of a cardiocyte (muscle) cell with two identical electrodes: one signal electrode inserted inside the cell, and one reference electrode outside in the extracellular medium, far away from the cell. The ion concentrations inside and outside the cell are given in the table below. At rest (equilibrium) the cell membrane is equally permeable to all three ion types, whereas at the peak of the action potential the cell membrane is permeable to Na^+ only. The GHK equation is:

$$V_m = \frac{RT}{F} \ln(10)\ \log_{10}\left(\frac{P_{\text{Na}}[\text{Na}^+]_o + P_{\text{K}}[\text{K}^+]_o + P_{\text{Cl}}[\text{Cl}^-]_i}{P_{\text{Na}}[\text{Na}^+]_i + P_{\text{K}}[\text{K}^+]_i + P_{\text{Cl}}[\text{Cl}^-]_o}\right)$$

and at room temperature $RT/F\ \ln(10) \approx 60$ mV.

	Inside	Outside
Na^+	10nM	100nM
K^+	100nM	10nM
Cl^-	10nM	100nM

(a) [10 pts] Find the voltage measured by the signal electrode relative to the reference electrode when the cell is at rest.

All permeabilities are equal at rest:

$$V_m = \frac{RT}{F} \ln(10) \log_{10}\left(\frac{[Na^+]_o + [K^+]_o + [Cl^-]_i}{[Na^+]_i + [K^+]_i + [Cl^-]_o}\right)$$

$$= \underbrace{\frac{RT}{F} \ln(10)}_{60\text{ mV}} \log_{10}\left(\underbrace{\frac{120\text{ nM}}{210\text{ nM}}}_{\frac{4}{7}}\right) = -14.6\text{ mV}$$

(b) [5 pts] Find the voltage measured by the signal electrode relative to the reference electrode when the cell is at the peak of its action potential.

Only Na+ is permeable during the action potential:

$$V_m = \frac{RT}{F} \ln(10) \cdot \log_{10}\left(\frac{[Na^+]_o}{[Na^+]_i}\right) \quad \text{(Na+ Nernst potential)}$$

$$= 60 \text{ mV} \cdot \underbrace{\log_{10}\left(\underbrace{\frac{100 \text{ mM}}{10 \text{ mM}}}_{=10}\right)}_{=1} = +60 \text{ mV}$$

(c) [5 pts] Why is it essential that the two electrodes are made of the same material? What happens to the measurement otherwise? Explain.

Such that the half-potentials cancel.

Otherwise, the difference in half-cell potentials between the two electrode materials will appear as a voltage offset in the measurement.

(d) **Bonus** [+10 pts] How do the voltages in (a) and (b) change in the case of extracellular measurements where the signal electrode is now positioned outside of the cell, 1 mm away from the cell. You may assume that the Na^+ conductance of the cell membrane is $g_{Na} = 1\ \mu A/V$, and the extracellular conductivity is $\sigma = 0.1$ A/Vm.

(a) At rest, the net current through the membrane is zero, and the extracellular potential is everywhere zero, including 1 mm away from the cell.

(b) During the (upstroke of the) action potential, only Na+ current flows through the membrane.

Current sink into extracellular space, due to the Na+ current across the membrane into the cell:

$$I_s = -I_{Na^+} = -g_{Na}\left(E_{Na} - \underbrace{V_m}_{= E_{rest} = -14.6\ mV}\right)$$

at the onset of the action potential

Signal electrode voltage at 1 mm distance from the cell:

$$V = \frac{I_s}{4\pi\sigma r} = -\frac{g_{Na}(E_{Na} - E_{rest})}{4\pi\sigma r} = -\frac{10^{-6}\frac{A}{V}\ 0.0746\ V}{4\pi\ 0.1\frac{A}{Vm}\ 0.001 m}$$

$$= -60\ \mu V$$

The extracellular action potential is negative, and much smaller than the +60mV intracellular action potential.

Note: At the end of the action potential upstroke, the Na+ current is zero, and the extracellular potential reaches zero. During the downstroke, current flows outward of the cell, and the extracellular potential goes slightly positive.