BENG 186B Winter 2019

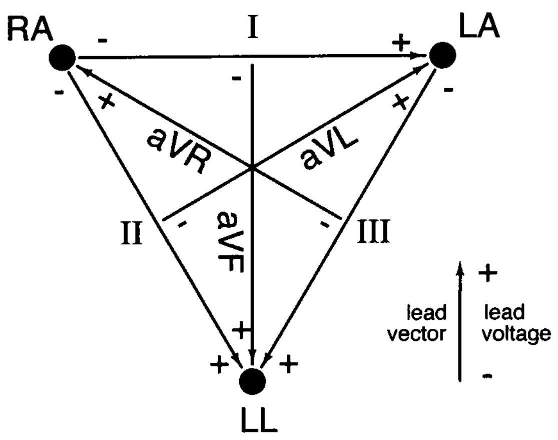
Quiz 3

Friday, March 8, 2019

Last Name, First Name: SOLUTIONS

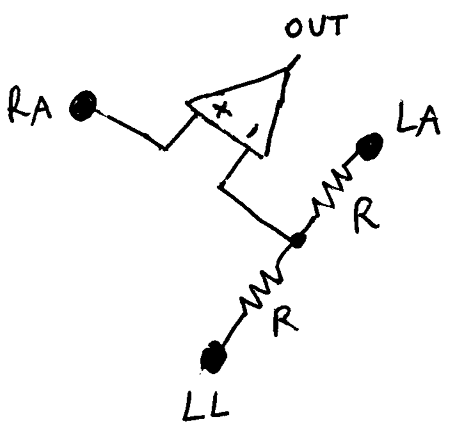
- This quiz is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- This quiz has 10 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- You have 50 minutes to complete this quiz.

1. **[20 pts]** An instrumentation amplifier is used to record the leads of the frontal electrocardiogram, as shown in the Einthoven's triangle below, by connecting its inputs to different combinations of the electrodes RA, LA, and LL. Assume the triangle is equilateral and the augmented lead vectors (aVR, aVL, and aVF) bisect the bipolar lead vectors (I, II, and III).



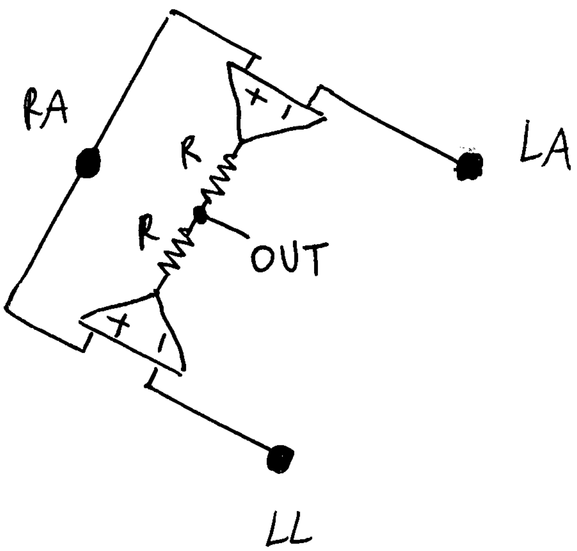
(a) [8 pts] In order to obtain the augmented lead aVR, show how to connect the electrodes to the non-inverting and inverting inputs of the instrumentation amplifier. In addition to the single amplifier, you may use any number of resistors in your network.

$$aVR = RA - \frac{1}{2}(LA + LL)$$



(b) [8 pts] Now you are given two instrumentation amplifiers, each with infinite input impedance and identical differential gain. Show how to obtain the same augmented lead aVR but now without drawing any current from the body. Again, you may use any number of resistors, but no additional amplifiers.

$$aVR = \frac{1}{2}(RA - LA) + \frac{1}{2}(RA - LL)$$



(c) [4 pts] Assume the cardiac vector points down vertically with magnitude 4 mV/m, and the distance between the shoulders is 40 cm. Find the voltage on lead I, and on lead aVF.

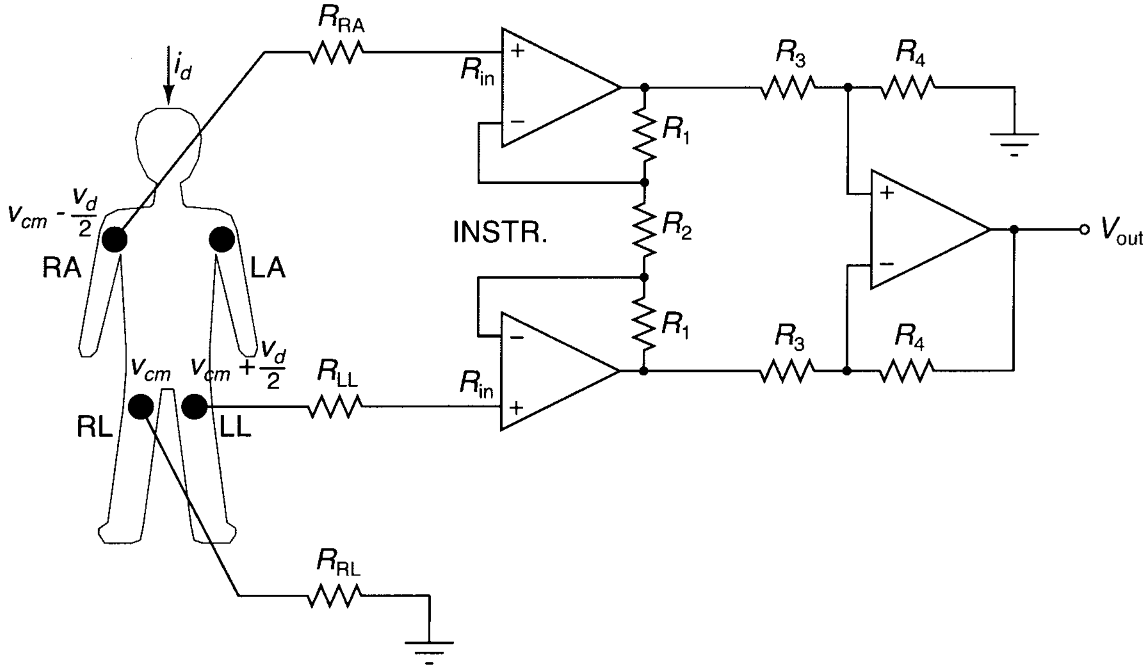
$$\vec{M} \perp \vec{I} \Rightarrow V_I = 0$$

$$\vec{M} \parallel \vec{aVF} \Rightarrow V_{aVF} = \vec{M} \cdot \vec{aVF}$$

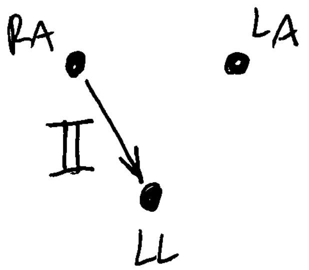
$$= 4\ mV/m \cdot \frac{\sqrt{3}}{2}\ 40\ cm$$

$$= 1.4\ mV$$

2. **[35 pts]** A two-stage instrumentation amplifier is connected to the body as shown below to record a single-lead electrocardiogram. The electrode-skin interface resistances are $R_{RA} = 130$ kΩ, $R_{LL} = 70$ kΩ, and $R_{RL} = 100$ kΩ. You may assume the opamps are ideal with infinite gain and infinite input impedance. The instrumentation resistances are $R_1 = 95$ kΩ, $R_2 = 10$ kΩ, $R_3 = 10$ kΩ, and $R_4 = 2$ MΩ, all with 1 % tolerance.



(a) [5 pts] What ECG lead does the IA output V_{out} represent, and with what voltage gain?



Lead II

$$A_d = \left(1 + 2\frac{R_1}{R_2}\right)\left(-\frac{R_4}{R_3}\right)$$

$$= 20 \cdot (-200) = -4{,}000$$

(b) [15 pts] 60 Hz line noise causes a displacement current of peak amplitude $i_d = 10\ \mu A$ to enter the body. Find the peak amplitude of the 60 Hz common-mode voltage v_{cm} in the body, and the resulting common-mode peak amplitude at the output of the instrumentation amplifier.

Peak voltages:

$$v_{cm} = R_{RL} \cdot i_d = 100\,k\Omega \cdot 10\,\mu A = 1\,V$$

$$v_{cm,out} = |A_c| \cdot v_{cm}$$

$$\leq \frac{1}{25} \frac{R_4}{R_3} \cdot v_{cm}$$ (worst case for ±1% mismatch in R_3, R_4)

$$= 8\, v_{cm}$$

$$= 8\,V$$ large!

(c) [5 pts] Despite your expectation in (b) you observe that the output of the instrumentation amplifier is a 60-Hz square wave with amplitude as large as the supply voltage. What is happening?

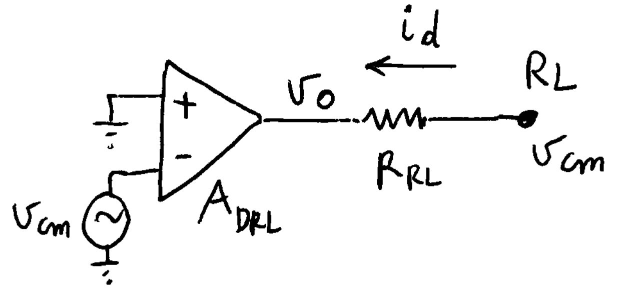
For a supply smaller than 8V, the second stage opamp SATURATES, with its output swinging between the supply rail voltages.

(d) [5 pts] You invest in more expensive laser-trimmed components for the resistances R_3 and R_4 now with 0.001 % tolerance, in the hope to reduce the common-mode swing at the output by a factor 1,000. However, you observe much less of a reduction, only by a factor 10, in the output common-mode at the 60 Hz line noise frequency. What could cause this to happen? *Hint*: Consider line capacitances.

Line capacitances load the inputs into the instrumentation amplifier, limiting the effective common-mode rejection:

$$\text{CMRR eff.} \leq \frac{|Z_{in}|}{|R_{RA} - R_{LL}|} = \frac{\frac{1}{j\, 2\pi 60\text{Hz}\, C_{line}}}{60\text{ k}\Omega}$$

(e) [5 pts] You decide to use active grounding to bring down the common-mode by an additional factor 100. For this purpose you insert a driven-right-leg (DRL) amplifier that drives the RL electrode with the amplified difference between the system ground and the body common-mode. Find the required voltage gain of the DRL amplifier.



$$v_o = -A_{DRL} \cdot v_{cm} = v_{cm} - R_{RL}\, i_d$$

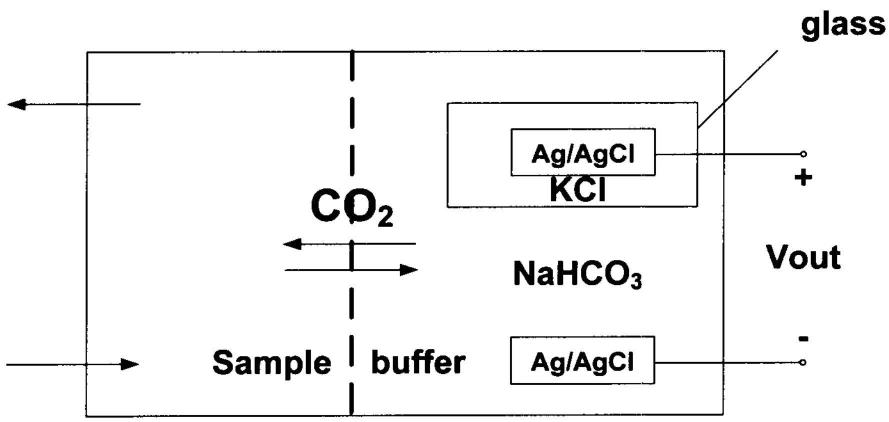
$$\Rightarrow v_{cm} = \frac{R_{RL}}{1 + A_{DRL}} \cdot i_d \quad \text{rather than } v_{cm} = R_{RL} \cdot i_d$$

$100\times$ reduction for $1 + A_{DRL} = 100$, or

$A_{DRL} = 99$

3. **[20 pts]** Consider the Severinghaus electrode shown below for measurement of PCO_2. The solution internal to the glass membrane is 0.15 mol/L KCl in pure water, and the glass membrane is permeable to H^+ only. The buffer contains 0.25 mol/L $NaHCO_3$, and is separated from the sample by a CO_2-permeable membrane. Assume $RT/F \ln(10) = 60$ mV at room temperature. The following equation may be useful:

$$H_2O + CO_2 \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$$



(a) [5 pts] You measure an output voltage V_{out} of 30 mV. What does that imply about the pH in the buffer solution?

$$V_{out} = 60\,\text{mV} \log_{10} \frac{[H^+]_{buffer}}{[H^+]_{KCl}}$$

$$= 60\,\text{mV} \cdot (pH_{KCl} - pH_{buffer}) = 30\,\text{mV}$$

$$pH_{KCl} = 7$$

$$\Rightarrow pH_{buffer} = 7 - \frac{30\,\text{mV}}{60\,\text{mV}} = 6.5$$

(b) [5 pts] Find the sensitivity of the voltage V_{out} to the partial pressure PCO_2.

$$V_{out} = 60\text{ mV} \cdot \log_{10}[H^+]_{buffer} + cst$$
$$= 60\text{ mV} \cdot \log_{10}[CO_2] + cst'$$
$$= 60\text{ mV} \cdot \log_{10} PCO_2 + cst''$$

$$\Rightarrow S = \frac{dV_{out}}{dPCO_2} = \frac{60\text{mV}}{\ln(10)} \cdot \frac{1}{PCO_2} = 25\text{mV} \frac{1}{PCO_2}$$

$$\frac{d}{dx}\log_{10}x = \frac{d}{dx}\frac{\ln x}{\ln(10)} = \frac{1}{\ln(10)} \cdot \frac{1}{x}$$

(c) [5 pts] The concentration of $NaHCO_3$ in buffer solution is now doubled. How does that affect the output voltage V_{out}? Explain.

$$V_{out} = 60\text{mV} \cdot \log_{10}[H^+]_{buffer} + cst = -60\text{mV} \log_{10}[HCO_3^-]_{buffer} + cst''$$

$\Rightarrow$ Doubling of $NaHCO_3$ lowers V_{out} by

$$-60\text{mV} \log_{10}(2) = -60\text{mV} \cdot 0.3 = -18\text{ mV}$$

(d) [5 pts] What is the voltage difference across the CO_2-permeable membrane between the sample and buffer solutions? Explain.

Zero: CO_2 is charge neutral, so there is no Nernst potential.

4. **[25 pts]** Circle the **best answer (only one answer per question)**:

(a) [4 pts] The current through the channel of an ISFET depends on:

i. Drain-to-source voltage
ii. Channel length
iii. Channel width
iv. Concentration of the bioanalyte
v. All of the above

(b) [4 pts] The inertance of a fluid-filled catheter system depends on:

i. Inner diameter of the catheter tube
ii. Length of the catheter tube
iii. Cross-section area of the catheter tube
iv. Mass density of the fluid
v. All of the above

(c) [4 pts] For accurate measurement of time-varying and local pressure in a blood vessel the instrument of choice is:

i. A micro-tipped manometer
ii. A mercury filled tube
iii. A sphygmomanometer
iv. A potentiostat
v. None of the above

(d) [4 pts] A driven right leg (DRL) system serves to:

i. Lower the effective impedance between the body and the system ground
ii. Actively ground the body potential
iii. Lower the common-mode potential variations across the body
iv. Lower the common-mode disturbance at the output of the instrumentation amplifier
v. All of the above

(e) [9 pts] Indicate for each statement below whether it is true or false:

i. **TRUE** / **FALSE**: Currents may enter the body without physical contact to external sources of noise.

ii. **TRUE** / **FALSE**: The CMRR of an instrumentation amplifier contributes directly to the SNR at its output.

iii. **TRUE** / **FALSE**: External fluid-filled catheter-based pressure transducers measure blood pressure directly with uniformly flat high-frequency response while being inexpensive and reusable.

iv. **TRUE** / **FALSE**: The electromagnetic flowmeter is highly invasive requiring electrodes placed on the vessel wall.

v. **TRUE** / **FALSE**: A Doppler flowmeter transduces total cardiac output into frequency differences in the return of ultrasonic waves impinging on the body.

vi. **TRUE** / **FALSE**: The Ag/AgCl electrode in the Severinghaus PCO_2 sensor gets consumed over time.

vii. **TRUE** / **FALSE**: A "resistive T" can be used to increase the sensitivity of a Clark PO_2 sensor.

viii. **TRUE** / **FALSE**: The IMFET is fabricated by immobilizing antibodies on the gate of a field-effect transistor.

ix. **TRUE** / **FALSE**: SO_2 can be measured optically and non-invasively with a skin-mounted sensor.