BENG 186B Winter 2020

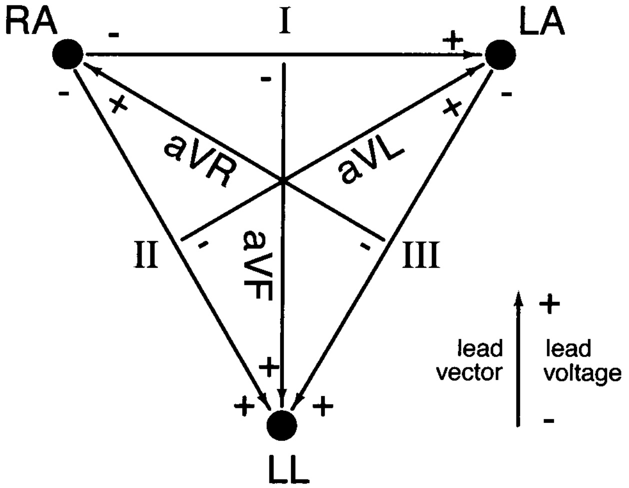
Quiz 3

Friday, March 6, 2020

Last Name, First Name: SOLUTIONS

- This quiz is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- This quiz has 11 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- The quiz takes 50 minutes to complete.

1. **[20 pts]** Consider Einthoven's triangle of the frontal electrocardiogram (ECG) for the three electrodes RA, LA and LL shown below. The triangle is equilateral and the augmented lead vectors (aVR, aVL, and aVF) bisect the bipolar lead vectors (I, II, and III).



(a) [8 pts] You are given an instrument that outputs the six-lead frontal ECG. However, four of the leads are malfunctioning, and only leads II and III provide useful outputs. Reconstruct the missing leads of the frontal ECG from the lead II and lead III outputs:

I = II − III

aVR = $\frac{1}{2}$ III − II

aVL = $\frac{1}{2}$ II − III

aVF = $\frac{1}{2}$ II + $\frac{1}{2}$ III

(b) [2 pts] Can you reconstruct the transversal ECG leads V1 through V6 from the leads II and III as well? Why?

No. Two leads in the frontal plane only span the frontal plane, not the transversal plane.

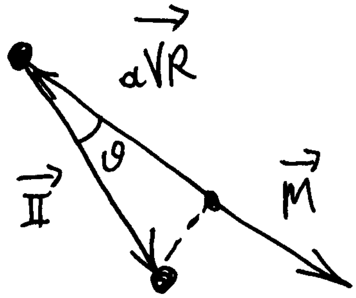
(c) [10 pts] You find that lead II measures 1 mV, and lead III measures zero. Estimate the direction and magnitude of the cardiac vector in the frontal plane. You may assume the distance between shoulders of the subject is roughly 58 cm.

$$V_{III} = \vec{M} \cdot \vec{III} = 0$$

$$\Rightarrow \vec{M} \perp \vec{III} \Rightarrow \vec{M} \parallel \vec{aVR}$$

$$V_{II} = \vec{M} \cdot \vec{II} = 1\,mV$$

$$\Rightarrow |\vec{M}| \cdot |\vec{II}| \cdot \underbrace{\cos(\underbrace{\theta}_{30^\circ})}_{\frac{\sqrt{3}}{2}} = 1\,mV$$



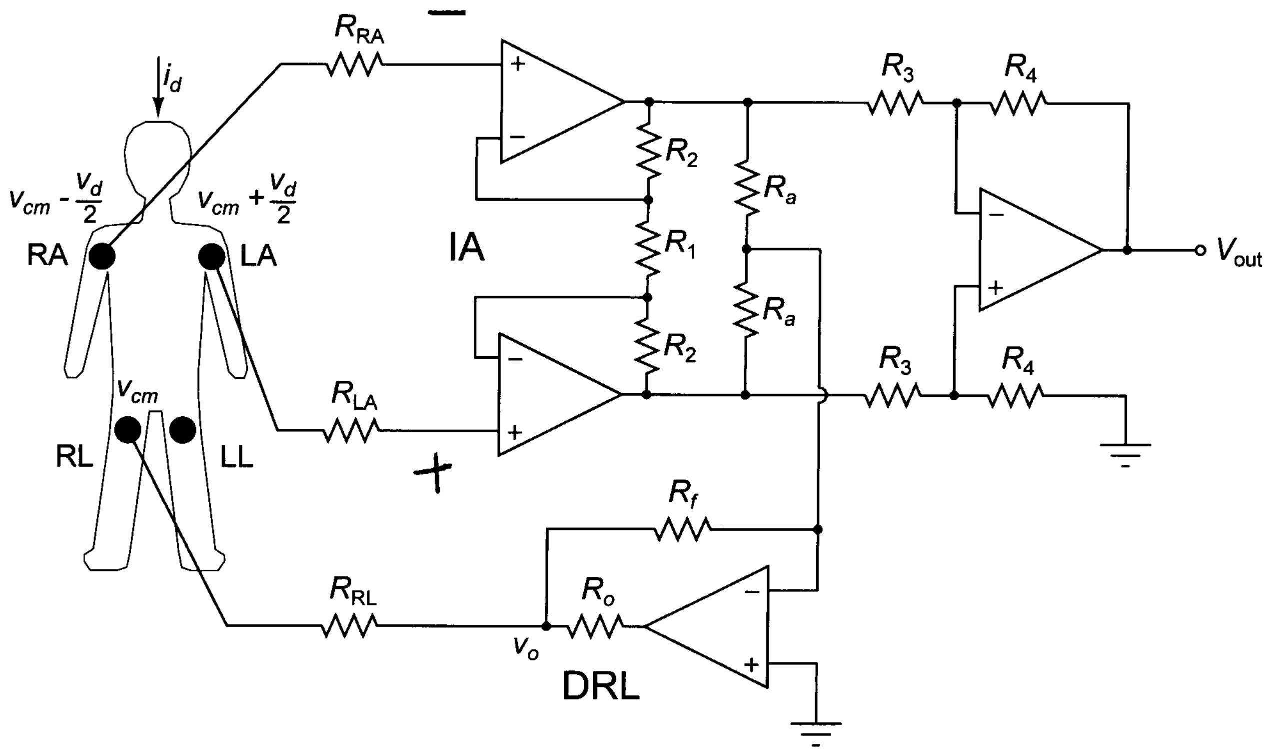
$$|\vec{II}| = |\vec{I}| = 58\,cm$$

$\Rightarrow \vec{M}$ points in the direction of $-\vec{aVR}$

with magnitude $|\vec{M}| = \dfrac{1\,mV}{|\vec{II}| \cdot \cos(\theta)}$

$$= \frac{1\,mV}{0.58\,m \;\; 0.87} = 2\,\frac{mV}{m}$$

2. **[35 pts]** An instrumentation amplifier (IA) and driven right leg (DRL) amplifier are connected as shown below to record a single-lead electrocardiogram while actively grounding the body. The electrode-skin interface resistances are $R_{RA} = 110\ \text{k}\Omega$, $R_{LA} = 90\ \text{k}\Omega$, and $R_{RL} = 100\ \text{k}\Omega$. All opamps are ideal with infinite input impedance. The IA resistances are $R_1 = 2\ \text{k}\Omega$ $R_2 = 199\ \text{k}\Omega$, $R_3 = 1\ \text{k}\Omega$ and $R_4 = 50\ \text{k}\Omega$, all with 1 % tolerance. The DRL resistances are $R_a = 20\ \text{k}\Omega$, $R_f = 1\ \text{M}\Omega$, and $R_o = 10\ \text{M}\Omega$.



(a) [5 pts] What ECG lead does the IA output V_{out} represent, and with what voltage gain?

Lead I

$$A_d = \left(1 + \frac{2R_2}{R_1}\right) \cdot \frac{R_4}{R_3} = (1 + 199) \cdot 50$$

$$= 10{,}000 \qquad (80\ \text{dB})$$

(b) [5 pts] Find the common-mode rejection ratio (CMRR) of the IA.

$$\text{CMRR} = \frac{A_d}{A_c} = \frac{\left(1+\frac{2R_2}{R_1}\right)\frac{R_4}{R_3}}{\frac{1}{25}\frac{R_4}{R_3}} = 25\left(1+\frac{2R_2}{R_1}\right)$$

$$= 5{,}000$$

(c) [5 pts] The overall CMRR of the system, including IA and electrodes, depends on a variety of factors, some of which are under control of the designer, and some not. How is CMRR affected by tighter tolerance on the IA resistances? And how is it affected at higher frequencies by capacitance to ground on the IA inputs?

CMRR of the IA is inversely proportional to resistance tolerance, so improves with tighter tolerance.

$$\text{CMRR}_{\text{eff}} \leq \frac{|Z_{in}|}{|R_{RA} - R_{LA}|} = \frac{\frac{1}{\omega C_{in}}}{20\ \text{k}\Omega}$$

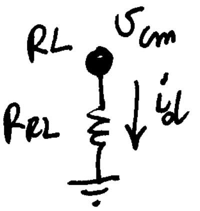
degrades with capacitance at higher frequencies

(d) [10 pts] For a displacement current i_d with 10 μA amplitude entering the body, find the amplitude of the common-mode voltage v_{cm} in the body, with the DRL connection as shown. How much improvement does the DRL active grounding offer over a passive ground connection to the RL electrode?

with DRL : $$v_o = -\frac{2R_f}{R_a} v_{cm} = v_{cm} - R_{RL} \cdot i_d$$

$$\Rightarrow \quad v_{cm} = \frac{R_{RL}}{1 + \frac{2R_f}{R_a}} i_d = \frac{100\,k\Omega}{1+100} \cdot 10\,\mu A \approx 10\,mV$$

without DRL : $$v_{cm} = R_{RL} \cdot i_d = 100\,k\Omega \cdot 10\,\mu A = 1\,V$$



"passive" ground

$\Rightarrow$ DRL active grounding improves v_{cm} by a factor 101 (roughly hundred-fold; the DRL gain)

(e) [5 pts] The amplitude of the differential signal v_d on the ECG lead is 100 μV. Find the corresponding signal-to-noise ratio (SNR) at the output of the instrumentation amplifier.

$$SNR_{out} = CMRR \cdot \frac{v_d}{v_{cm}}$$

$$= 5{,}000 \cdot \frac{100\,\mu V}{10\,mV}$$

$$= 50 \quad (34\ dB)$$

(f) [5 pts] What is the function of the resistance R_o in the DRL feedback loop? Explain.

Current limiting for short-circuit protection.

$$\text{Maximum current} = \frac{V_{supply}}{R_o}$$ where

V_{supply} is the opamp supply voltage (either polarity)

3. **[20 pts]** Consider the PO_2 sensor below consisting of a Clark electrode and a transimpedance amplifier (TIA). The flow rate ϕ of the sample through the chamber is maintained constant at 1 mL/s, the solution in the buffer is 0.1 mol/L KCl, and the values of TIA resistances are $R_1 = R_3 = 1\ \mathrm{M\Omega}$ and $R_2 = 1\ \mathrm{k\Omega}$. The following equations may be useful:

$$O_2 + 2\,H_2O + 4\,e^- \rightleftharpoons 2\,H_2O_2 + 4\,e^- \rightleftharpoons 4\,(OH)^-$$

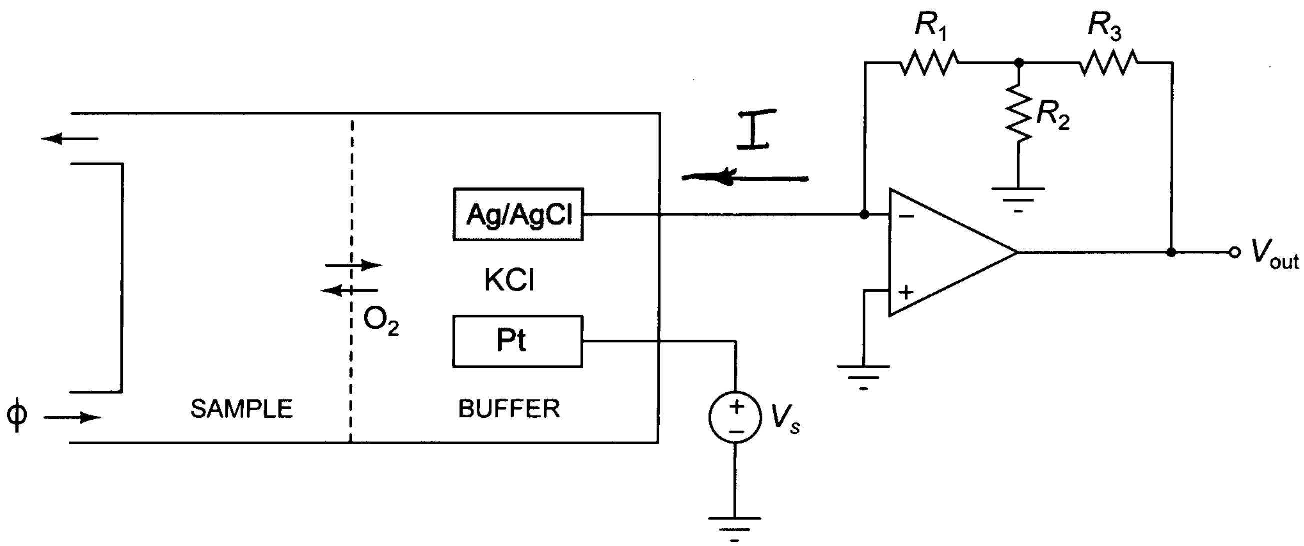
$$4\,(OH)^- + 4\,KCl \rightleftharpoons 4\,KOH + 4\,Cl^-$$

$$4\,Ag + 4\,Cl^- \rightleftharpoons 4\,AgCl + 4\,e^-$$

$$I = 4\,F\,[O_2]\,\phi$$

where $F = 96{,}485$ C/mol is the Faraday constant.

$$R_{eff} = \frac{R_1 R_3}{R_2}$$



(a) [5 pts] What output voltage V_{out} do you expect when the voltage source V_s is set to zero? Explain.

Zero. No electrons are injected by the Pt electrode into the solution, so the current I is zero.

(b) [10 pts] Now with the voltage source V_s set to -0.7 V, find the sensitivity of the voltage output V_{out} to oxygen concentration $[O_2]$ in the sample at steady-state.

$$V_{out} = R_{eff} \cdot I = \frac{R_1 R_3}{R_2} 4F [O_2] \phi$$

$$\Rightarrow S = \frac{dV_{out}}{d[O_2]} = \frac{R_1 R_3}{R_2} 4F \phi$$

$$= \underbrace{\frac{1M\Omega \cdot 1M\Omega}{1k\Omega}}_{10^9 \Omega} \cdot 4 \cdot 96{,}485 \frac{C}{mol} \cdot 10^{-3} \frac{L}{s}$$

$$= 4 \, 10^{11} \frac{V L}{mol}$$

(c) [5 pts] Explain how the Ag/AgCl electrode gets consumed with the consumption of oxygen.

For every mol of O_2, 4 moles of Ag get converted into AgCl.

4. **[25 pts]** Circle the **best answer (only one answer per question)**:

   (a) [4 pts] The damping in the frequency response of the external fluid-filled catheter tube depends on:

      i. Mass density of the fluid
      ii. Viscosity of the fluid
      iii. Air in the fluid
      iv. All of the above
      v. None of the above

   (b) [4 pts] A Doppler flowmeter measures:

      i. Cardiac output
      ii. Velocity of blood flow
      iii. Oxygenation of hemoglobin in the blood stream
      iv. All of the above
      v. None of the above

   (c) [4 pts] The Severinghaus electrode:

      i. Measures concentration of carbon dioxide in a sample
      ii. Transduces PCO_2 into pH
      iii. Measures the Nernst potential across a hydrogen-permeable membrane
      iv. All of the above
      v. None of the above

   (d) [4 pts] An immunologically sensitive field-effect transistor:

      i. Is a special type of ion-sensitive field-effect transistor
      ii. Measures optical fluorescence resulting from antigen-antibody binding in a label-free preparation
      iii. Transduces concentration of antigens or antibodies in the solution to capacitance in a Wheatstone bridge
      iv. All of the above
      v. None of the above

(e) [9 pts] Indicate for each statement below whether it is true or false:

i. **TRUE / FALSE**: The electric field for the ECG current dipole falls off as one over distance, $1/r$. ($1/r^3$)

ii. **TRUE / FALSE**: Shielding of electrical wires in a bioinstrument reduces interference.

iii. **TRUE / FALSE**: The inertance of fluid within a rigid pipe in a hydraulic system can be modeled by a resistor as an electric analog. (inductance)

iv. **TRUE / FALSE**: The sphygmomanometer gives a direct, instantaneous measure of blood pressure. (indirect, peak)

v. **TRUE / FALSE**: Indicator-dilution methods give indirect measures of cardiac output.

vi. **TRUE / FALSE**: The electromagnetic flowmeter transduces blood velocity into magnetic field. (into electric field by static magnetic field)

vii. **TRUE / FALSE**: PO_2 can be inferred from measurement of SO_2 with knowledge of pH and temperature. (to some extent, when not saturated)

viii. **TRUE / FALSE**: A buffered solution of HCl maintains constant pH for a stable reference in a pH sensor.

ix. **TRUE / FALSE**: Beer's law relates total optical absorbance to specific absorptivity of different states of hemoglobin.