BENG 186B Winter 2021

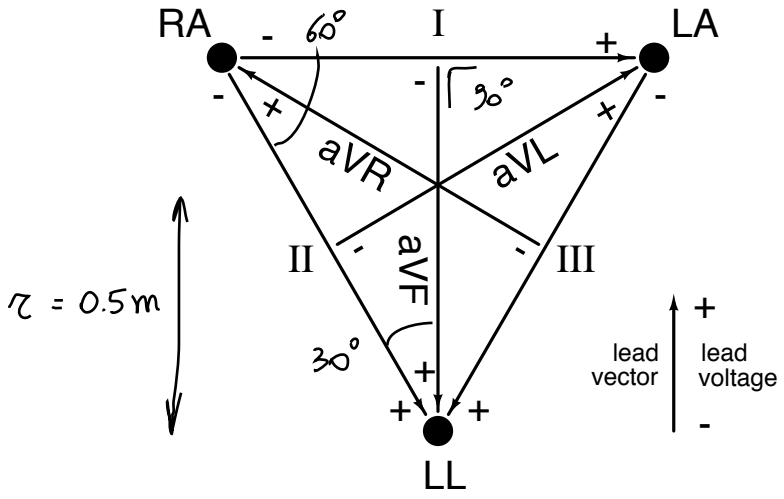
Quiz 3

Friday, March 5, 2021

Last Name, First Name: SOLUTIONS

- This quiz is on-line, open-book, and open-notes. You may use a calculator or an equivalent program, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due March 5, 2021 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any quiz-related material among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[20 pts]** Consider Einthoven's triangle of the frontal electrocardiogram (ECG) shown below. The triangle is equilateral and the augmented lead vectors (aVR, aVL, and aVF) bisect the bipolar lead vectors (I, II, and III). All three electrodes RA, LA and LL are at distance $r = 50$ cm to the heart.



(a) [6 pts] At the peak of the R wave, the cardiac vector points vertically downwards with magnitude 5 mV/m. Find the voltage on lead I, lead II, and lead aVF.

$$V(\mathrm{I}) = \vec{M} \cdot \vec{\mathrm{I}} = 0 \qquad (\vec{M} \perp \vec{I})$$

$$V(\mathrm{II}) = \vec{M} \cdot \vec{\mathrm{II}} = |\vec{M}| \cdot |\vec{\mathrm{II}}| \cdot \cos(30^\circ)$$

$$= 5\,\frac{\mathrm{mV}}{\mathrm{m}} \cdot \sqrt{3}\ 0.5\,\mathrm{m} \cdot \frac{\sqrt{3}}{2} = 3.75\ \mathrm{mV}$$

$$V(\mathrm{aVF}) = \vec{M} \cdot \vec{\mathrm{aVF}} = |\vec{M}| \cdot |\vec{\mathrm{aVF}}| \qquad (\vec{M} \parallel \vec{\mathrm{aVF}})$$

$$= 5\,\frac{\mathrm{mV}}{\mathrm{m}} \cdot \frac{3}{2}\,0.5\,\mathrm{m} = 3.75\ \mathrm{mV}$$

(b) [6 pts] At the peak of the S wave, the cardiac vector now points horizontally from right to left with magnitude 1 mV/m. Find the voltage on lead I, lead II, and lead aVF.

$$V(\mathrm{I}) = \vec{M} \cdot \vec{I} = |\vec{M}| \cdot |\vec{I}| \quad (\vec{M} \parallel \vec{I})$$

$$= 1 \frac{mV}{m} \cdot \sqrt{3}\ 0.5\,m = 0.866\ mV$$

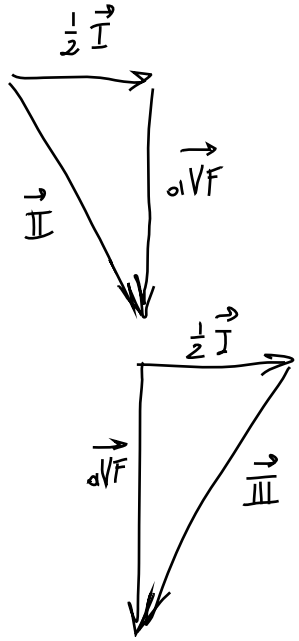
$$V(\mathrm{II}) = \vec{M} \cdot \vec{II} = |\vec{M}| \cdot |\vec{II}| \cdot \cos(60^\circ)$$

$$= 1 \frac{mV}{m} \cdot \sqrt{3}\ 0.5\,m \cdot \frac{1}{2} = 0.433\ mV$$

$$V(aVF) = \vec{M} \cdot \overrightarrow{aVF} = 0 \quad (\vec{M} \perp \overrightarrow{aVF})$$

(c) [8 pts] Show that measurement of just leads I and aVF is sufficient to reconstruct the full 6-lead frontal electrocardiogram. Write the reconstructed leads II, III, aVR, and aVL in terms of the measured leads I and aVF. Why is it useful to the cardiologist to have all 6 frontal leads when only 2 of them are sufficient to completely characterize the cardiac vector in the frontal plane?

Leads I and aVF are orthogonal and constitute a complete basis in the frontal plane. Any vector in the frontal plane can be written as a linear combination of these two.



$$\vec{II} = \frac{1}{2}\vec{I} + \vec{aVF}$$

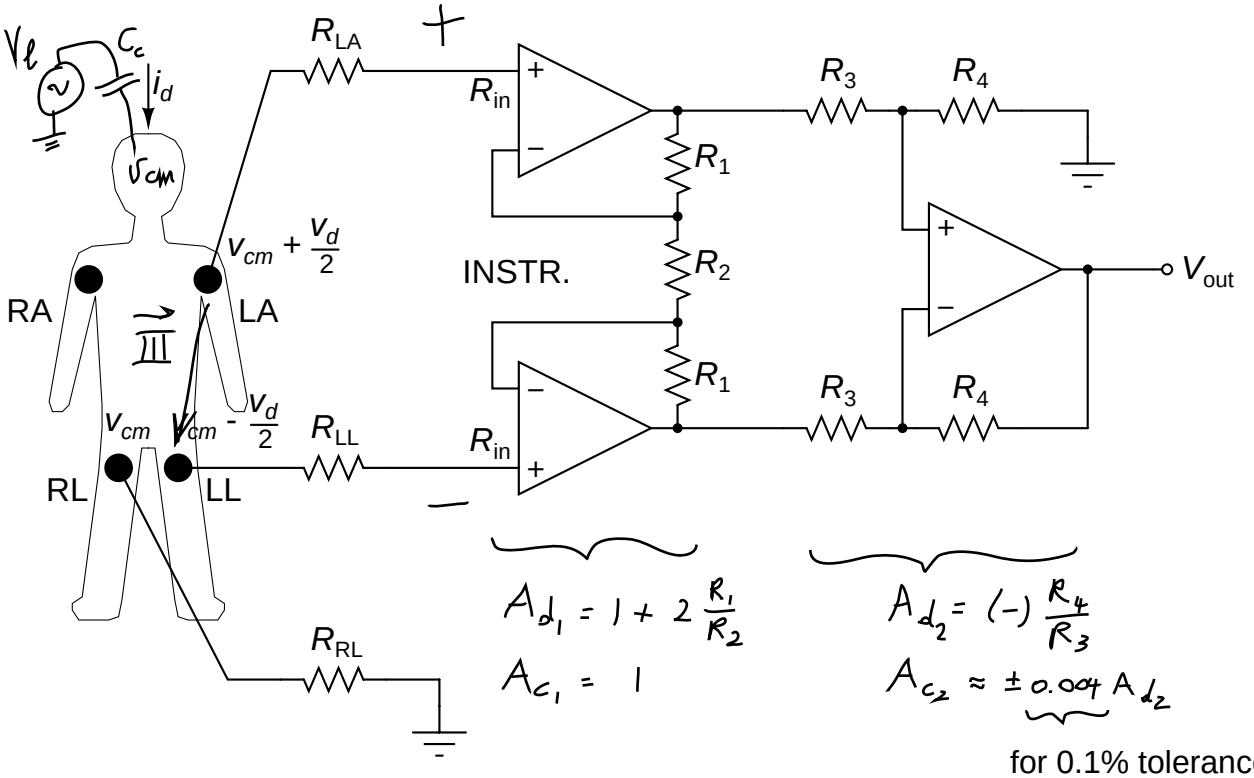
$$\vec{III} = -\frac{1}{2}\vec{I} + \vec{aVF}$$

$$\vec{aVR} = -\frac{1}{2}\left(\vec{I} + \vec{II}\right) = -\frac{3}{4}\vec{I} - \frac{1}{2}\vec{aVF}$$

$$\vec{aVL} = \frac{1}{2}\left(\vec{I} - \vec{III}\right) = \frac{3}{4}\vec{I} - \frac{1}{2}\vec{aVF}$$

The 6 frontal leads, and their complements, project the cardiac vector at all angles with 30 degree increments. The lead with minimum voltage amplitude identifies the direction most orthogonal to the cardiac vector with 30 degree accuracy.

2. **[35 pts]** A two-stage instrumentation amplifier (IA) is connected to the body as shown below to record a single-lead electrocardiogram. The electrode-skin interface resistances are $R_{LA} = 80$ kΩ, $R_{LL} = 120$ kΩ, and $R_{RL} = 100$ kΩ. The opamps are ideal with infinite gain and infinite input impedance ($R_{in} = \infty$). The IA resistances are $R_1 = 99.5$ kΩ, $R_2 = 1$ kΩ, $R_3 = 10$ kΩ, and $R_4 = 100$ kΩ, all with 0.1 % tolerance.



(a) [5 pts] What ECG lead does the IA output V_{out} represent, and with what voltage gain?

Lead III with voltage gain $-\left(1 + 2\frac{R_1}{R_2}\right)\frac{R_4}{R_3} = -200 \times 10 = -2000$

Equivalently, lead -III with voltage gain +2000

(b) [10 pts] A nearby 33 kVrms 60 Hz AC power line couples with 1 pF capacitance to the body. Find the resulting rms amplitude of the displacement current i_d and common-mode voltage v_{cm} in the body.

$$i_d = C_c \frac{dV_\ell}{dt} = j\omega C_c V_\ell \quad \text{(Week 6)}$$

$$i_{d\,rms} = \omega C_c V_{\ell\,rms} = 2\pi\ 60\,Hz\ \ 1\,pF\ \ 33\,kV_{rms} = 12.4\,\mu A_{rms}$$

(rms: root mean square; or standard deviation)

$$v_{cm\,rms} = R_{RL} \cdot i_{d\,rms} = 100\,k\Omega \cdot 12.4\,\mu A_{rms} = 1.24\,V_{rms}$$

(c) [10 pts] Find the common-mode rejection ratio (CMRR) of the IA, and the resulting common-mode rms amplitude at the IA output. How does it change when accounting for 20 pF parasitic capacitance at each input of the IA?

$$CMRR_{IA} = \frac{|A_d|}{|A_c|} = \left(1 + 2\frac{R_1}{R_2}\right) \cdot \underbrace{\frac{1}{0.004}}_{=250} = 50{,}000 \quad (94\ dB)$$

0.1% tolerance!

$$V_{out\ rms} = A_c\, v_{cm\ rms} = 0.004\,\frac{R_4}{R_3}\, v_{cm\ rms} = 50\ mV_{rms}$$

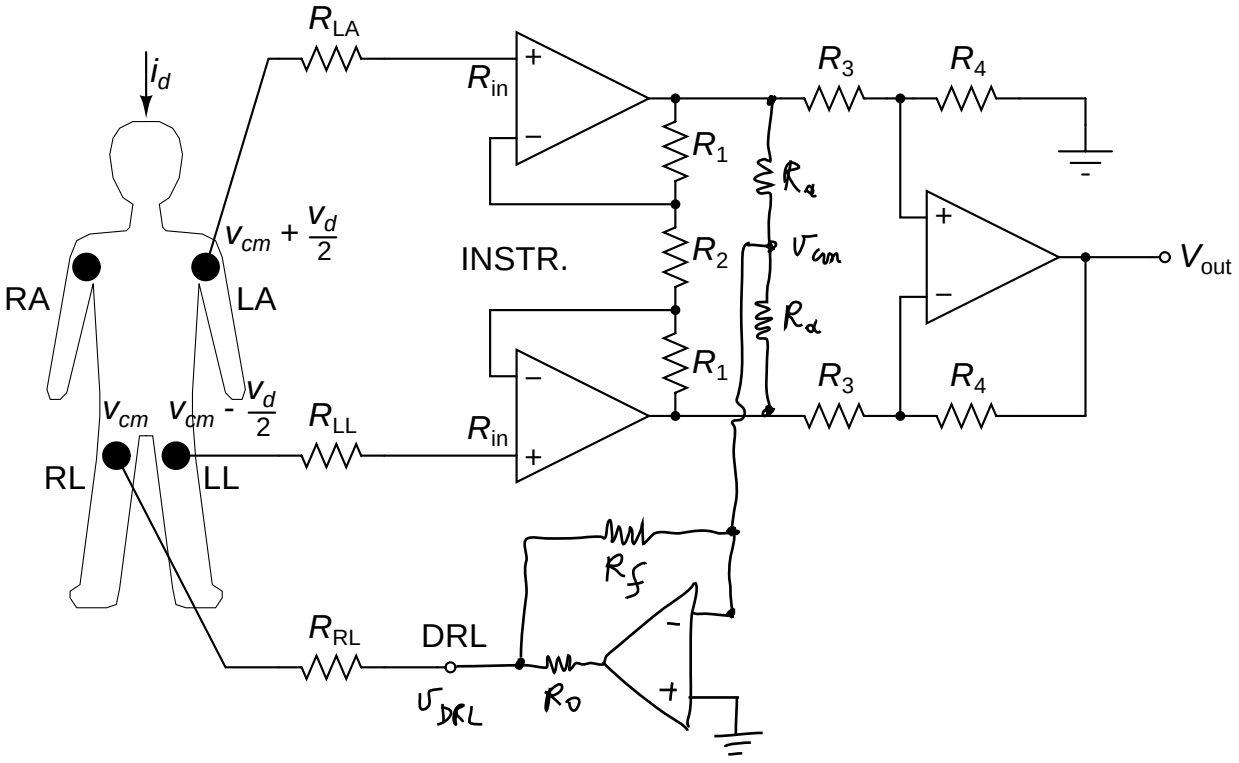
That is just for the IA by itself. The effective CMRR of the overall system is dominated by electrode impedance mismatch considering finite input impedance due to the parasitic input capacitance:

$$CMRR_{eff} \approx \frac{|Z_{in}|}{|R_{LA} - R_{LL}|} = \frac{\left|\frac{1}{jwC_{in}}\right|}{|R_{LA} - R_{LL}|} = \frac{\frac{1}{2\pi\, 60Hz\, 20pF}}{40\ k\Omega}$$

$$= 3{,}316 \quad (70.4\ dB)$$

$$V_{out\ rms} \approx \frac{|A_d|}{CMRR_{eff}}\, v_{cm\ rms} = \frac{2{,}000}{3{,}316}\, 1.24\ V_{rms} = 750\ mV_{rms}$$

(d) [10 pts] You decide to use active grounding to bring down the common-mode noise by an additional factor 1,000. For this purpose you insert a driven-right-leg (DRL) amplifier that drives the RL electrode with the amplified difference between the system ground and the body common-mode. Show your design of the DRL amplifier interfacing to the IA and the body, and indicate values for all components used.



Passive grounding: $v_{cm\,rms} = R_{RL}\, i_{d\,rms}$

Active grounding: $v_{cm\,rms} = \dfrac{R_{RL}}{1 + 2\frac{R_f}{R_a}}\, i_{d\,rms}$ (Week 6; Lecture 12)

$\Rightarrow \quad 1 + 2\frac{R_f}{R_a} = 1000$ for a thousand-fold reduction

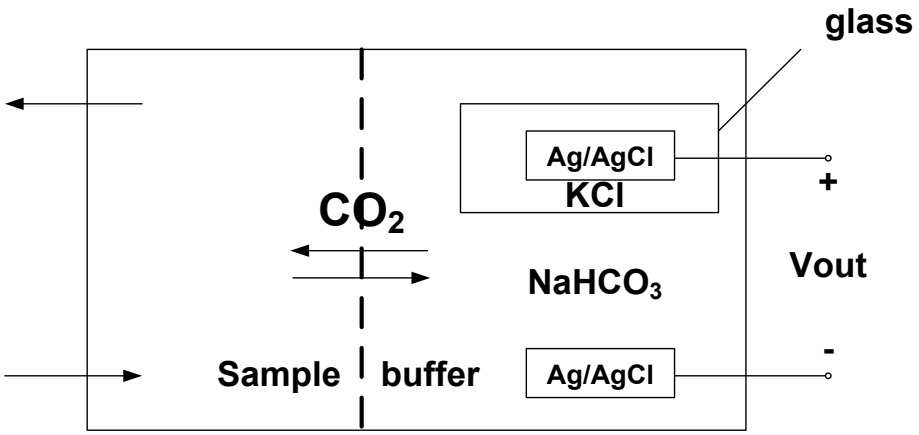
e.g. $R_a = 2\,k\Omega$

$R_f = 1\,M\Omega$

Also: $R_0 = 1\,M\Omega$ (optional, desirable for protection purposes)

3. **[20 pts]** Consider the Severinghaus electrode shown below for measurement of PCO_2. The solution internal to the glass membrane is 0.1 mol/L KCl in pure water, and the glass membrane is permeable to H^+ only. The buffer contains a saturated solution of $NaHCO_3$, and is separated from the sample by a CO_2-permeable membrane. Assume $RT/F \ln(10) = 60$ mV at room temperature. The following equation may be useful:

$$H_2O + CO_2 \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$$



(a) [5 pts] You measure an output voltage V_{out} of 60 mV. What does that imply about the pH in the buffer solution?

$$V_{out} = \frac{RT}{F} \ln(10) \log_{10} \frac{[H^+]_{buffer}}{[H^+]_{KCl}}$$

$$= 60 \text{ mV} \left(pH_{KCl} - pH_{buffer} \right) = 60 \text{ mV}$$

$$\Rightarrow pH_{buffer} = pH_{KCl} - 1 = 6$$

$= 7$

(Pure salt is pH neutral.)

(b) [10 pts] The concentration of $NaHCO_3$ in the buffer solution is adjusted so that the output voltage is zero for a calibration sample of known partial pressure $PCO_{2cal} = 1$ kPa. Find the partial pressure PCO_2 of a blood sample in terms of the output voltage V_{out} measured for that sample.

$$\log_{10} PCO_2 = \frac{V_{out}}{60\,mV} + cst$$

a constant (see Lecture 15)

$$\log_{10} PCO_{2\,cal} = \frac{0}{60\,mV} + cst$$

same constant!

$$\Rightarrow PCO_2 = PCO_{2\,cal} \cdot 10^{\frac{V_{out}}{60\,mV}}$$

(c) [5 pts] Why is it necessary to have the blood go through the sample chamber at non-zero flow rate? How is the measurement affected when the blood sample has a fixed finite volume?

The sample needs to pass through the chamber at non-zero flow rate because there needs to be enough supply of CO2 in the sample so that its concentration does not get diluted by the supply already in the buffer solution. Otherwise, for a sample of limited volume, its CO2 concentration will mix with that of a previous sample.

4. **[25 pts]** Circle the **best answer (only one answer per question)**:

(a) [4 pts] The Clark electrode measures:

i. Carbon dioxide concentration
ii. Carbon dioxide flow rate
iii. Oxygen consumption
iv. Hydrogen peroxide concentration
v. None of the above

(b) [4 pts] The resistance of a fluid-filled catheter depends on:

i. Inner diameter of the catheter tube
ii. Length of the catheter tube
iii. Cross-section area of the catheter tube
iv. Viscosity of the fluid
v. All of the above

(c) [4 pts] For quick measurement of systolic and diastolic pressure the instrument of choice is:

i. A micro-tipped manometer
ii. A sphygmomanometer
iii. A tonometer
iv. A strain gauge
v. None of the above

(d) [4 pts] A Doppler flowmeter has greatest sensitivity when:

i. the ultrasonic transmitter and receiver are perpendicular to the blood vessel
ii. the ultrasonic receiver is parallel to the blood vessel
iii. the frequency of the sound approaches zero
iv. the transducer is separated from the body
v. All of the above

(e) [9 pts] Indicate for each statement below whether it is true or false:

i. **TRUE** / **FALSE**: The Wilson Central Terminal is obtained by placing an electrode directly on the heart.

ii. **TRUE** / **FALSE**: The CMRR of an instrumentation amplifier is limited by its input impedance.

iii. **TRUE** / **FALSE**: Compliance of air in a fluid filled catheter decreases its bandwidth and increases its damping.

iv. **TRUE** / **FALSE**: The electromagnetic flowmeter requires a DC magnetic field for measurement of blood velocity.

v. **TRUE** / **FALSE**: A resistive T allows to increase transresistance for highly sensitive amperometric measurement.

vi. **TRUE** / **FALSE**: The Ag/AgCl electrode in the Clark PO_2 sensor gets consumed over time.

vii. **TRUE** / **FALSE**: Indicator-dilution methods for cardiac output measurement are completely non-invasive.

viii. **TRUE** / **FALSE**: An IMFET is a special type of ISFET.

ix. **TRUE** / **FALSE**: SO_2 can be measured non-invasively in a wristmounted sensor.