

Neuromorphic Integrated Bioelectronics - Fall 2025

BENG 216, UC San Diego

Homework 3: Due October 24

The objective of the third homework is to design a circuit emulating the dynamics of a cochlear channel. Here we abstract the function of the transduction of sound pressure through acoustic waves along the basilar membrane to electrical impulses by the outer hair cells to that of bandpass filtering of the acoustic waveform followed by rectification and lowpass filtering to arrive at a measure of instantaneous sound intensity in a frequency band. We implement these functions using transconductance-capacitance (g_m - C) analog continuous-time linear filters, along with a conventional bridge rectifier circuit. This homework has no layout component and is purely a design and simulation effort, using the same tools and methods for schematic capture and simulation that you used for homework 1.

1. *Transconductance-capacitance analog continuous-time linear filters* [40 points].

We start by considering g_m - C linear filters composed of wide-range differential operational transconductance amplifiers (OTA) driving purely capacitive loads, implementing lowpass and bandpass filter characteristics. The wide-range OTA used here is the same one described in R. Lyon and C.A. Mead, "Electronic Cochlea," IEEE Trans. Acoustics, Speech, and Signal Processing (ASSP), vol. 36 (7), 1988.

- (a) Show, from first principles (the equations describing current-voltage relationship of nMOS and pMOS transistors in subthreshold and in saturation, ignoring drain conductance) that for a wide-range OTA with input nMOS differential pair at bias current I_b and output through current mirrors with unity gain, the output current I_{out} is given by

$$I_{out} = I_b \tanh \left(\frac{1}{2} \frac{\kappa_n}{V_{th}} (V_{in}^+ - V_{in}^-) \right) \quad (1)$$

where κ_n is the gate coupling efficiency of the nMOS transistors in the differential pair, V_{th} is the thermal voltage kT/q , and V_{in}^+ and V_{in}^- are the non-inverting and inverting voltage inputs to the differential pair. Continue to show that, for sufficiently small magnitude in differential voltage $V_{in} = V_{in}^+ - V_{in}^-$, the transconductance g_m of the OTA is given by

$$g_m = \left. \frac{dI_{out}}{dV_{in}} \right|_{V_{in}=0} = \frac{1}{2} \frac{\kappa_n}{V_{th}} I_b \quad (2)$$

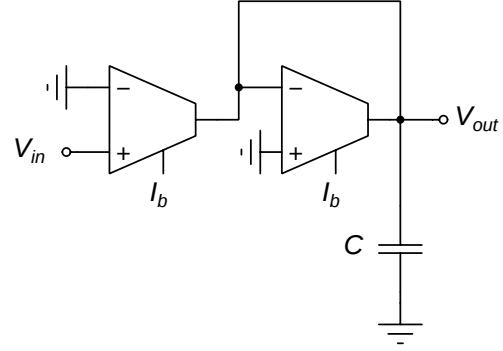
directly proportional to bias current.

- (b) Now consider a first-order section lowpass filter constructed using two wide-range OTAs with bias current I_b , and one capacitance C , as depicted on the right. Show that the frequency response of this filter circuit is given by the following transfer function

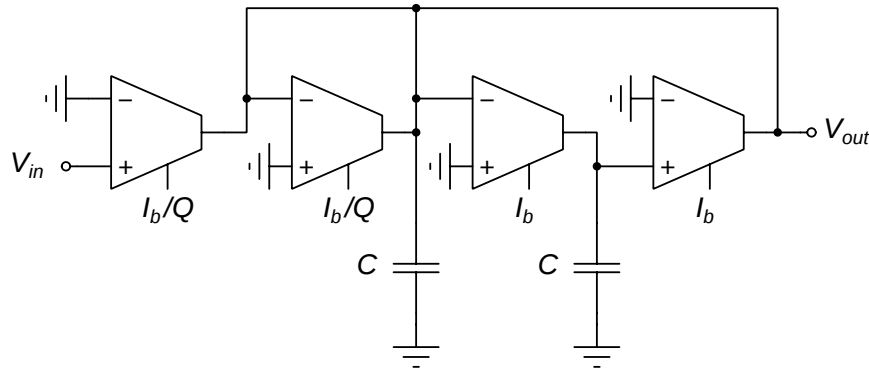
$$H_{LP}(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + \frac{j\omega}{\omega_c}} \quad (3)$$

where ω is radial frequency (in radians per second, or $2\pi f$ with frequency f in cycles per second or Hz), and ω_c is the cut-off radial frequency. Express ω_c in terms of the physical parameters of the circuit.

Show a Bode plot of the transfer function (log magnitude and phase of $H_{LP}(j\omega)$) as a function of ω on a log scale, see e.g. <https://www.mathworks.com/help/ident/ref/dynamicsystem.bode.html> for $C = 1$ pF, $\kappa_n = 0.75$, $V_{th} = 25$ mV (at room temperature), and for the following three bias currents: $I_b = 10$ pA, 100 pA, and 1 nA. Interpret your results.



- (c) Next consider a second-order bandpass filter constructed as a “biquad” with four wide-range OTAs and two capacitances C , as shown below. The leftmost two OTAs in the biquad structure are given a bias current I_b / Q , whereas the rightmost two are biased at I_b , where Q is a positive “quality” factor. Find the transfer function $H_{BP}(j\omega) = V_{out}(j\omega) / V_{in}(j\omega)$ of the filter in terms of the physical parameters of the circuit. Show the Bode plots of the transfer function for the same parameters as in 1. (b), and for the following two values of the quality factor: $Q = 1$, and 10. What does the quality factor signify in the bandpass filter response?

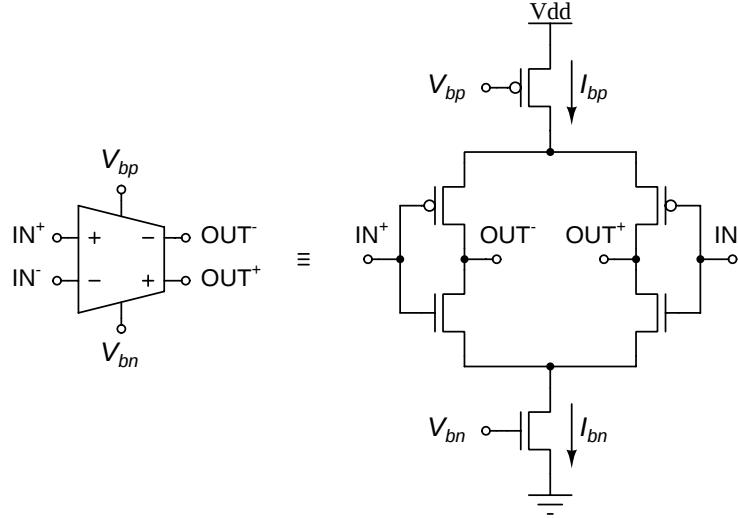


2. Fully differential, current-reuse, OTA and g_m - C filter design [30 points].

Here we study a variation on the OTA design for greater energy efficiency and noise robustness, that achieves greater transconductance for the same total amount of current consumption by reusing the same bias current twice with nMOS and pMOS differential pairs in tandem, and that mitigates common-mode disturbances in supply variations and noise coupling by employing all fully differential signals in the data path.

- (a) Below is a simplified diagram of a CMOS circuit implementing a fully differential current-reuse OTA consisting of a stack of two complementary nMOS and pMOS differential pairs, driven

by bias currents I_{bn} and I_{bp} respectively. Nominally, the two bias currents are identical, set to a common I_b bias current through current mirrors (not shown). The complementary stack of differential pairs operates in a differential push-pull configuration: the nMOS differential pair pulls current from the output nodes whereas the pMOS differential pair pushes equal current into the output nodes, but on opposite sides. In what follows it is assumed that nMOS and pMOS transistors have identical gate coupling efficiencies $\kappa = \kappa_n = \kappa_p$.



Show that for balanced bias currents, $I_{bn} = I_{bp} = I_b$, the output current is balanced and approximately differential in the input voltage:

$$I_{out}^+ = -I_{out}^- = I_b \tanh\left(\frac{1}{2} \frac{\kappa}{V_{th}} (V_{in}^+ - V_{in}^-)\right) \quad (4)$$

and continue to show that this yields twice more transconductance for the same bias current:

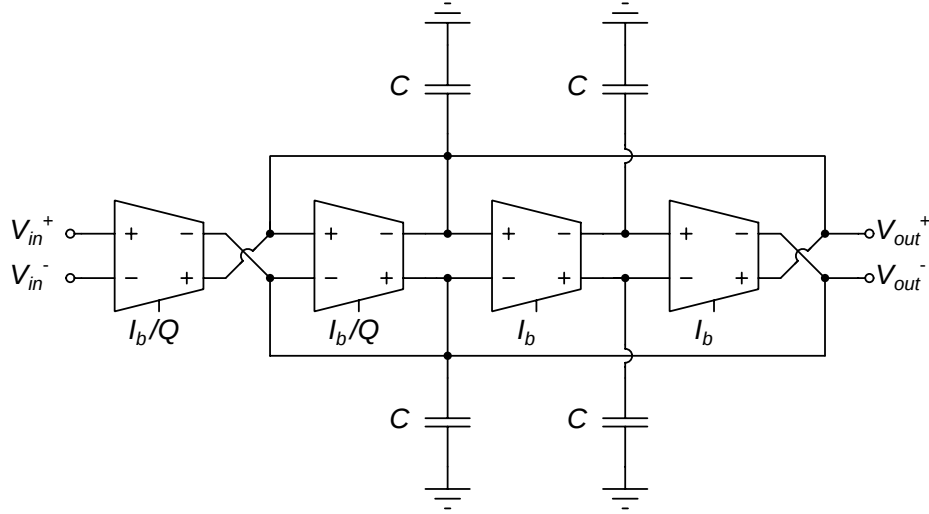
$$g_m = \left. \frac{dI_{out}}{dV_{in}} \right|_{V_{in}=0} = \frac{\kappa}{V_{th}} I_b \quad (5)$$

where $I_{out} = I_{out}^+ - I_{out}^- = 2I_{out}^+$ is the differential output current for the differential input voltage $V_{in} = V_{in}^+ - V_{in}^-$.

What happens when I_{bn} and I_{bp} differ? And what might cause these two currents to differ despite being generated from the same bias current I_b through current mirrors? Explain.

BONUS [Extra 10 points]: Come up with a way to mitigate any such effects on current imbalance between I_{bn} and I_{bp} , by dynamically biasing one of these two currents in order to maintain the common mode of the output $\frac{1}{2}(V_{out}^+ + V_{out}^-)$ at a constant mid-level voltage at all times. The methodology to accomplish this is called common-mode feedback control in the analog integrated circuit design community.

- (b) We now consider a fully differential extension of the above biquad bandpass filter shown below, which uses the current-reuse fully differential OTA rather than the single-ended wide-range OTA to maintain a fully differential signal format along the data path.



Implement the circuit in the 130nm Skywater process using the `nfet_1v8` model for the nMOS transistors and the `pfet_1v8` model for the pMOS transistors in the `sky130_fd_pr_main` core library. All transistors are sized with width $W = 1 \mu\text{m}$, and with length $L = 1 \mu\text{m}$ for the current bias transistors, and $L = 180 \text{ nm}$ for the differential pair transistors. The supply voltage is $V_{\text{dd}} = 1.2 \text{ V}$, and the voltage inputs V_{in}^+ and V_{in}^- are centered around mid-level common-mode voltage $V_{CM} = 0.6 \text{ V}$ as the DC operating point. Using Maestro, run an AC sweep simulation of the circuit, around the common-mode DC level at the input, for the following parameters: $C = 1 \text{ pF}$, $I_b = 10 \text{ pA}$, and $Q = 10$. Compare your simulation results with what you obtained for the behavioral model in 1. (c), and explain any differences that you observe.

3. Design of one frequency channel of a silicon cochlea [30 points].

Finally, we proceed with the complete design of a single frequency channel of a simplified silicon model of the mammalian cochlea, using the filter elements that you have just characterized, and some additional components. Specifically, implement the schematic of your design as a cascade of the following functional blocks:

- Two cascaded stages of second-order bandpass filter biquads with fully differential current-reuse OTAs, as studied in 2. (b);
- Full-wave rectifier receiving the cascaded bandpass output. You could implement the full-wave rectifier using a standard 4-diode bridge topology (e.g., , https://en.wikipedia.org/wiki/Diode_bridge), using diode-connected MOS transistors (e.g., gate-to-drain shorted pMOS transistors with low threshold, and/or with n-well bulk connected to the source) instead of p-n junction diodes, or any alternative circuit topology of your choice that produces a differential output voltage proportional to the absolute value of the differential input voltage;
- A single first-order section lowpass filter with fully differential current-reuse OTAs, and with the same level of bias current I_b / Q and same capacitance C as in the biquads.

Use the same supply and common-mode voltage levels, and same transistor sizing, as in 2. (b). A single current I_b supplied to the circuit should generate two sets of I_{bn} and I_{bp} bias gate voltages: one set for the I_b OTAs, and the other for the I_b / Q OTAs, where the quality factor Q is fixed in hardware to a value of 10.

Set the bias current I_b to implement a 1 kHz center frequency for the bandpass response, and verify the lowpass-filtered rectified bandpass response of the silicon cochlea model by observing the voltage output of your circuit for:

- (a) A constant-amplitude 100 mVpp (millivolts peak-to-peak) voltage input at 1 kHz frequency;
- (b) A constant-amplitude 100 mVpp frequency-modulated (FM) voltage input at 1 kHz center frequency, with 10% modulation depth at 10 Hz frequency (i.e., the instantaneous frequency is $f = f_0 + 0.1 f_0 \sin(2\pi f_m t)$ where $f_0 = 1$ kHz and $f_m = 10$ Hz).

Explain what you observe.

Submission Guidelines: You are encouraged to work on teams and exchange solution strategies and share configuration of the EDA tools, but you must complete the homework yourself, and are not allowed to copy other's work. In particular you cannot share schematics capture and simulations for homework submission and need to complete these entirely by yourself. It is anticipated that no two independent schematic entries are identical.

Turn in your homework as a single PDF over canvas by the due date. Scanned handwritten notes are fine, including the printout or screenshots of the design materials.