Winnerless Competition Principle in Individual Neurons and Cognitive Networks

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Abstract

Winnerless competition is a theoretical model used to study cognitive and attention-based processes using the sequential switching between metastable states of a neurodynamical system. This competition is observed in both physical neural networks and in higher-ordered cognition. I demonstrate this principle in a network of three interconnected Fitzhugh-Nagumo neurons and in the working memory model, a more abstract cognitive processes using competitive Lotka-Volterra dynamics.

1 Introduction

1.1 Background

Neural systems provide an example of a highly integrated network able to adapt to external stimuli, respond quickly and accurately to external information, and decide future courses of action. One of the main goals of studying these systems is to understand the link between the brain itself and how it is projected to cognition. Consequently, one of the major questions is: what are the mechanisms that transform the extremely complex, noisy, and many-dimensional brain activity into low-dimensional and predictable cognitive behavior?

It is important to understand the difference between brain dynamics and cognition (mind dynamics). Brain dynamics involves physical spatial-based regions of the brain and the neurons that compose them. Because neurons are often noisy due to intrinsic ion fluctuations and synaptic-based connections, the dynamical equations that model these processes involve noise as well. Cognition, on the other hand, is a noiseless process that is the result of the spatio-temporal organization of global brain activity. One main goal is to answer the following: what is the dynamical process by which the brain is able to suppress both intrinsic and stimuli-based noise to proceed with cognitive processes in a noiseless fashion? For example, suppose one is skiing and sees a tree. How is it that the brain is able to ignore noisy stimuli (cold temperature, falling snow, etc.) and very quickly focus attention onto the specific task at hand: avoiding the tree?

We therefore provide an insight into this process using a principle known as winnerless competition, a sequential switching among metastable states of neuronal systems.

1.2 Winnerless Competition Principle

The state space of the model is formed from a finite number of variables which are functions of time. Each variable constitutes a different cognitive modality. The mathematical image for transient information processes is that of a chain of metastable states represented by saddle fixed points in phase space connected by a one-dimensional unstable separatrix. Under specific conditions, the trajectories that enter the vicinity of the chain become trapped and cannot leave. This topological structure is known as the Stable Heteroclinic Channel [1].

Focusing attention requires inhibitory networks in the brain to help block incoming stimuli that are unrelated to a specific task. Multiple regions of the brain form these networks [2]-[4], and as such, I investigate the dynamical mechanism behind different brain structures participating in cooperative and competitive activity. The competitive dynamics, which leads to a model to describe attentional modes, can be modeled with the simplest Lotka-Volterra type equations to characterize dynamical behavior. We also demonstrate this principle with a Fitzhugh-Nagumo network model of three neurons connected in sequence.

2 The Models

2.1 Fitzhugh-Nagumo

The Fitzhugh-Nagumo model [5] is a simplification of the Hodgkin-Huxley model which retains the features required to ensure basic neuronal behavior, namely switching between spiking bursts and resting states. Let us call x_i the membrane potential of the i^{th} neuron. The membrane voltage thus follows the differential equation:

$$\frac{dx_i}{dt} = \frac{1}{\tau_1}(x_i - \frac{1}{3}x_i^3 - y_i + S_i)$$

S_i is the input current corresponding to an external stimuli and may be split into two parts: one part which is the same for each neuron of the system (a baseline excitatory current), and a part which is specific to the i^{th} neuron from information input to the system. The second variable necessary to display spiking is denoted as y_i and is governed by the following equation:

$$\frac{dy_i}{dt} = x_i - by_i + a$$

Here, a and b are parameters between 0 and 1. Because the switching from bursting to resting state and back is due to neural inhibition between neurons, we introduced one more variable to account for inhibitory connections among the neurons in the system, obeying the differential equation:

$$\frac{dz_i}{dt} = \frac{1}{\tau_2}(\sum_j g_{ji} G(x_j) - z_i)$$

Here, j refers to the other neurons in the system and g_{ji} is the strength of synaptic inhibition of the neuron i by the neuron j. The simplest choice for G is a step function given by:

$$G(x) = \begin{cases} 0, & x \le 0 \\ 1, & x > 0 \end{cases}$$

Finally, we add a $-z_i(x_i - v_{min})$ term to the first equation to force the membrane potential into a constant resting state of v_{min} (<0) whenever the inhibitory variable takes large values due to other neurons' spiking activities. Our final membrane potential equation becomes:

$$\frac{dx_i}{dt} = \frac{1}{\tau_1}(x_i - \frac{1}{3}x_i^3 - y_i + S_i - z_i(x_i - v_{\min}))$$

The following initial conditions and parameters were used:

$$\begin{cases} x_o = -1.20 \\ y_o = -0.62 \\ z_o = \ 0.00 \end{cases}$$

$$\begin{cases} \tau_1 = 0.08 \\ v_{\min} = \ -1.5 \\ S = 0.35 \\ a = 0.7 \\ b = 0.8 \\ \tau_2 = 3.1 \\ g = 1 \end{cases}$$

2.2 Lotka-Volterra Model

In general, the intrinsic dynamics of these neurons are governed by many variables corresponding to ion channels and intracellular processes. Because we are only interested in the neural coding, we can simplify the model to study the activity level of neuron clusters using square-nonlinear competitive Lotka-Volterra dynamics:

$$\frac{da}{dt} = diag(a_1(t),\ldots,a_n(t))[b - Pa]$$

Here, each a_i is the variable that measures the activity level, b is a vector describing the excitation rate in the activity level of the neuron cluster, and P is an n-dimensional matrix that measures the strength of inhibition of the neurons among the clusters. Moreover, the trajectories tell us the behavior of sequential switching among neural clusters. This model has been used to study a wide range of cognitive processes, including working memory and attention-based dynamics [6].

Here, we use the example of working-memory to illustrate this concept. Suppose one was interested in memorizing a phone number. How does one go about this task? There are two possible routes: memorize each digit individually or "chunk" the number into sections. In either case, each bit of information takes up some cognitive resource, namely attention. The mathematical image of this is a chain of meta-stable states represented by saddle fixed points (Figure 1). Here, each saddle fixed point represents a chunk of information in short-term memory.

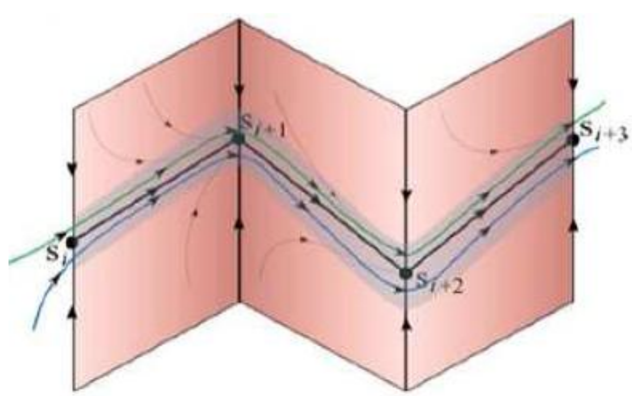

Figure 1. Sequence of meta-stable states in phase space with four saddle points and the one-dimensional unstable separatrix which connects them. Trajectories in the neighborhood of this chain stay within the vicinity of it showing the stability and robustness to noise.

It is easy to show that the origin is an unstable fixed point of the model. Furthermore, for a sequence of variables $a_1,...,a_n$ it is easy to show using linearization that the fixed point a_k^* = $(0,0,...,b_k,0,...,0)$ has eigenvalues (with normalization of self-inhibition parameters to 1):

$$\lambda_{k,k+1} = b_{k+1} - p_{k+1,k} b_k \qquad and \qquad \lambda_{k-1,k} = b_k - p_{k,k-1} b_{k-1}$$

where $\lambda_{k,k+1}$ is the eigenvalue associated with the k^{th} saddle point to the k+1 saddle point direction. Furthermore, it has been proven [7] that an n-dimensional competitive Lotka-olterra model always converges onto an n-1 dimensional topological structure known as a carrying simplex. Thus to study the long-term behavior of the neuron model, we need to only look at what happens to the trajectories on the simplex.

Finally, although cognition is noiseless, brain structures associated with sequential switching cognitive processes are not, and these structures can be modeled with the same Lotka-Volterra dynamics [8]. Thus, we subject the model to noise to understand the dynamical behavior of the model. Namely, we look at the following equation where the inhibition parameters are perturbed by a Gaussian white noise with a matrix σ controlling the scaling size of the noise:

$$\frac{da}{dt} = diag(a_1(t),...,a_n(t))[b - Pa + \sigma a dW(t)]$$

In fact, a recent theorem [9] show the following bound on the upper growth rate of the activity levels: Suppose $\sigma_{ii} > 0$ and $\sigma_{ij} \geq 0$ for all i,j. Then for any $b \in \mathbb{R}^n$, $P \in \mathbb{R}^{nxn}$, and $a_o(t) \in \mathbb{R}^+$:

$$\limsup \frac{\ln \sum_{i=1}^{n} a_i(t)}{\ln t} \leq 1$$

Thus, the long-term activity levels of these structures cannot grow faster in time than a linear relationship.

3 Results

Here we show simulations of both networks described above, both in the presence and absence of noise.

3.1 Fitzhugh-Nagumo Network

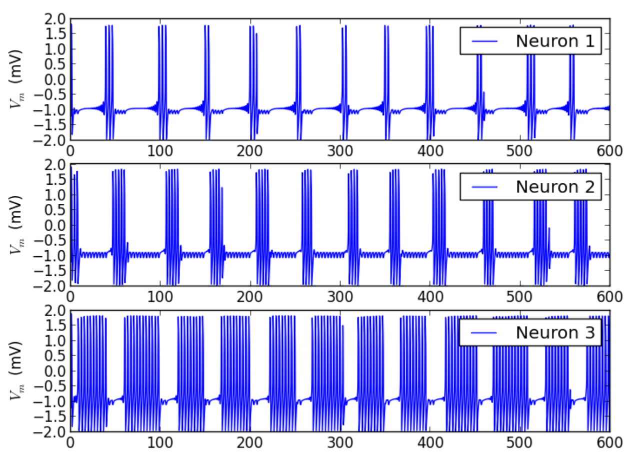

Figure 1a. Three Fitzhugh-Nagumo neurons connected in sequence without noise.

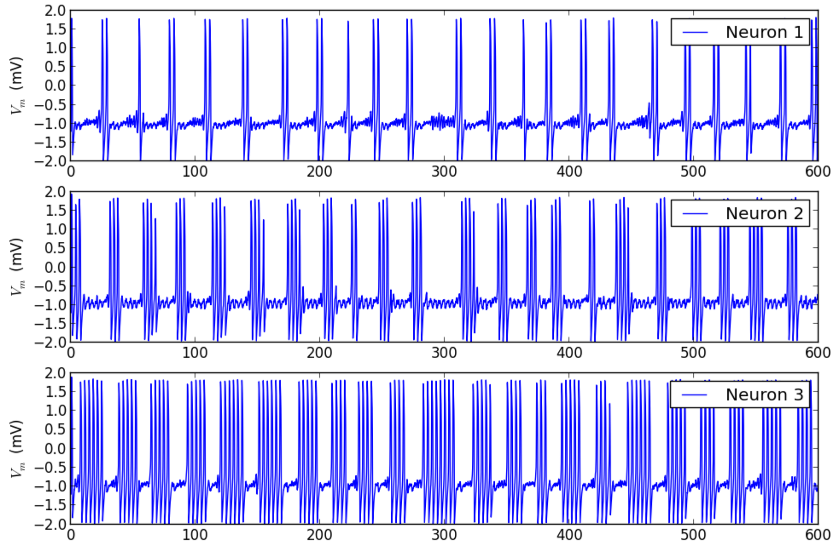

Figure 1b. Three Fitzhugh-Nagumo neurons connected in sequence with medium-low noise (white noise variance = 0.2)

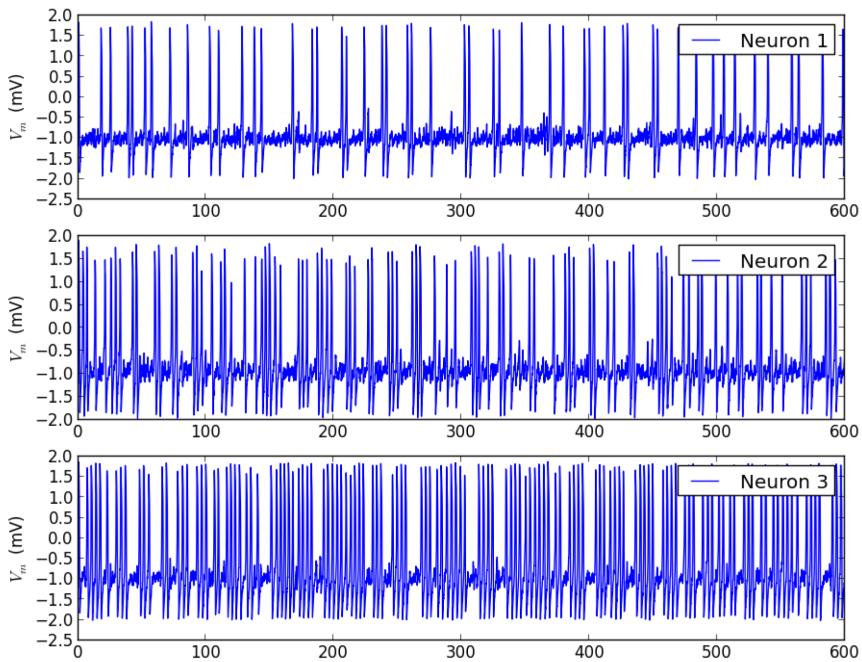

Figure 1c. Three Fitzhugh-Nagumo neurons connected in sequence with high noise (white noise variance = 0.7)

In the absence of noise, the three neurons switch between bursting states and resting states in a sequential order. In the presence of small enough noise, the neurons still continue to switch on and off in a winnerless competition fashion. However, increasing the noise appears to destroy the sequential switching. Thus, one may ask how large noise can be before sequential switching is gone? This question remains unanswered.

3.2 Mesoscale Cognitive Networks

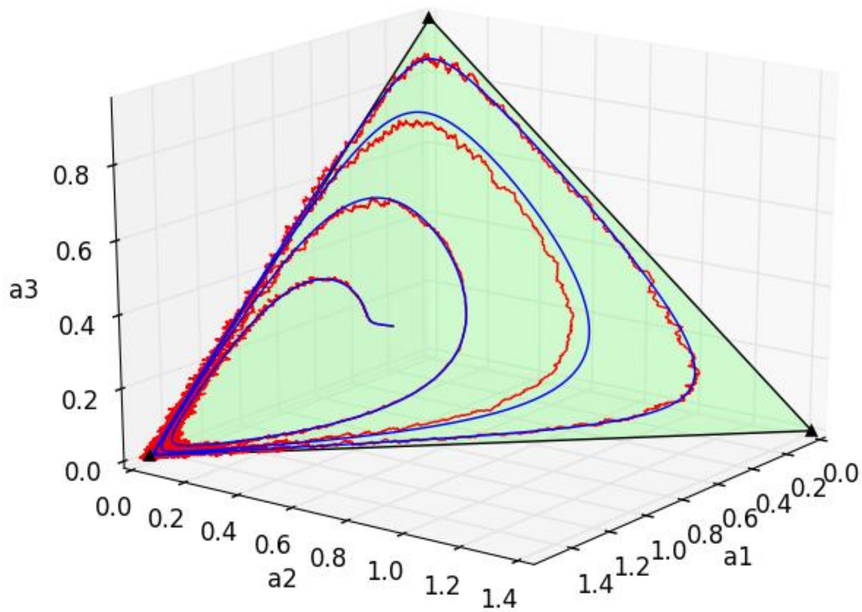

Figure 2. Trajectories of three-dimensional competitive Lotka-Volterra equations without noise (blue trajectory) and with Gaussian white noise (red trajectory). The green triangle is the two-dimensional carrying simplex where all trajectories regardless of initial conditions converge onto.

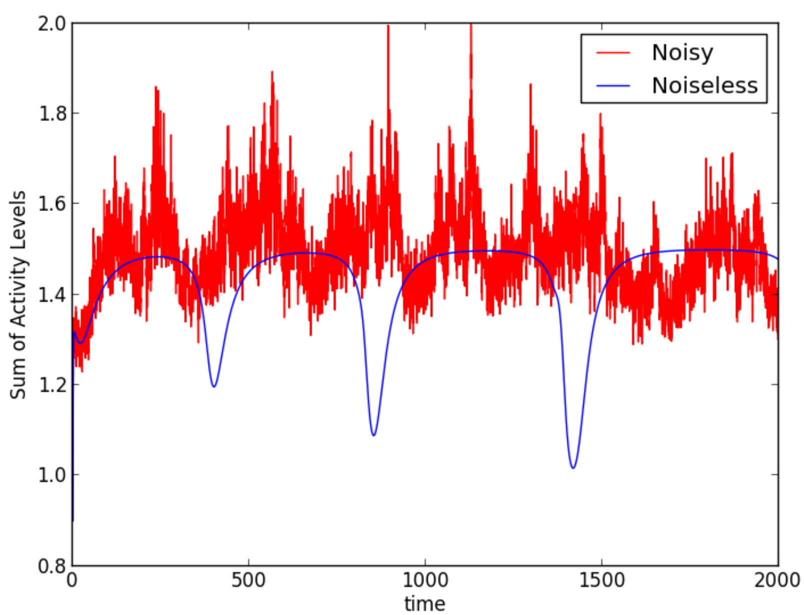

Figure 3. Plot of the sum of the activity levels over time to demonstrate the previously-listed supremum theorem and the maximum growth rate the activity levels can climb.

In the example shown above in Figure 2, Gaussian white noise still causes the trajectory to follow the general pattern as the unperturbed trajectory. Physically, this suggests that brain structures that are compete in winnerless competition and subjected to white noise still obey sequential switching as expected. Figure 3 (and the corresponding theorem) show that white noise does not cause any explosion in the activity levels of the brain structures. Thus, with regards to white noise, we have both analytical proof and numerical simulation that show how the corresponding activity levels of brain structures and neurons within them behave. What remains to be seen is how these structures behave when subjected to colored noise, whether or not sequential switching still occurs, and what the limit on the growth rate of the activity level is.

Acknowledgments

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