Neurodynamics

Computational Lab 4

1. Basic HH Model

Current Equations

$$\frac{dV}{dt} = \frac{1}{C}\,(-I_{Na} - I_K - I_L + I_{ext}) \quad (1)$$

$$I_{Na} = g_{Na}\, m^3 h\, (V - E_{Na}) \quad (2)$$

$$I_K = g_K\, n^4\, (V - E_K) \quad (3)$$

$$I_L = g_L\, (V - E_L) \quad (4)$$

Parameters

$$\begin{array}{rclrcl} C & = & 1\,\mu F/cm^2 & & & \\ E_{Na} & = & 45\,mV; & g_{Na} & = & 120\,mS/cm^2 \\ E_K & = & -82\,mV; & g_K & = & 36\,mS/cm^2 \\ E_L & = & -59.387\,mV; & g_L & = & 0.3\,mS/cm^2 \end{array} \quad (5)$$

Gating variable differential equations

$$\frac{dm}{dt} = \alpha_m(V)\,(1-m) - \beta_m(V)\,m \quad (6)$$

$$\frac{dh}{dt} = \alpha_h(V)\,(1-h) - \beta_h(V)\,h \quad (7)$$

$$\frac{dn}{dt} = \alpha_n(V)\,(1-n) - \beta_n(V)\,n \quad (8)$$

Gating variable nested equations

$$\alpha_m(V) = 0.1(V+45)/(1-\exp(-(V+45)/10)) \quad (9)$$

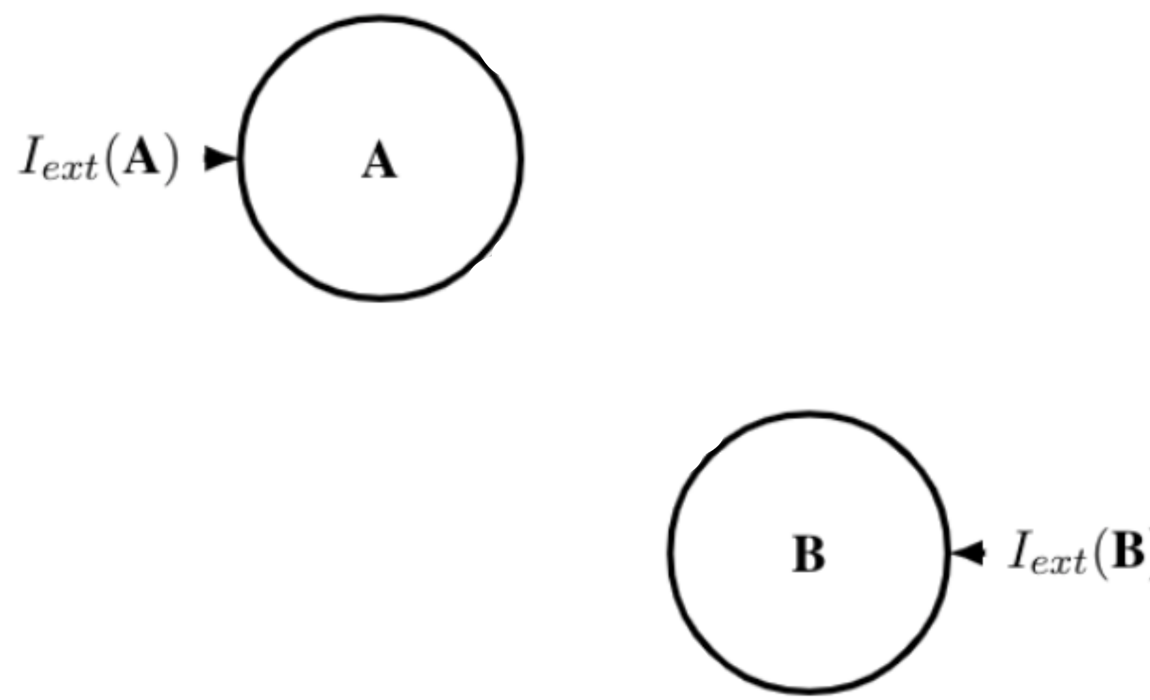
$$\beta_m(V) = 4\,\exp(-(V+70)/18) \quad (10)$$

$$\alpha_h(V) = 0.07\,\exp(-(V+70)/20) \quad (11)$$

$$\beta_h(V) = 1/(1+\exp((-(V+40))/10)) \quad (12)$$

$$\alpha_n(V) = 0.01(V+60)/(1-(\exp((-(V+60)/10)-1)) \quad (13)$$

$$\beta_n(V) = 0.125\,\exp(-(V+70)/80) \quad (14)$$



2. Add Inhibition

Current Equations

$$\frac{dV}{dt} = \frac{1}{C}\left(-I_{Na} - I_K - I_L + I_{ext}\right) \quad (1)$$

$$I_{Na} = g_{Na}\, m^3 h\, (V - E_{Na}) \quad (2)$$

$$I_K = g_K\, n^4\, (V - E_K) \quad (3)$$

$$I_L = g_L\, (V - E_L) \quad (4)$$

$$I_{syn} = g_{GABA_A}\, r\, (V_{post} - E_{Cl}) \quad (15)$$

Parameters

$$\begin{aligned} C &= 1\,\mu F/cm^2 \\ E_{Na} &= 45\,mV; \quad & g_{Na} &= 120\,mS/cm^2 \\ E_K &= -82\,mV; \quad & g_K &= 36\,mS/cm^2 \\ E_L &= -59.387\,mV; \quad & g_L &= 0.3\,mS/cm^2 \end{aligned} \quad (5)$$

$$\begin{aligned} E_{Cl} &= -80\,mV \\ \alpha_r &= 5\,mM^{-1}ms^{-1}; \quad & \beta_r &= 0.18\,ms^{-1} \\ [T]_{max} &= 1.5\,mM \\ K_p &= 5\,mV; \quad & V_p &= 7\,mV \end{aligned} \quad (18)$$

Gating variable differential equations

$$\frac{dm}{dt} = \alpha_m(V)\,(1-m) - \beta_m(V)\,m \quad (6)$$

$$\frac{dh}{dt} = \alpha_h(V)\,(1-h) - \beta_h(V)\,h \quad (7)$$

$$\frac{dn}{dt} = \alpha_n(V)\,(1-n) - \beta_n(V)\,n \quad (8)$$

$$\frac{dr}{dt} = \alpha_r[T]\,(1-r) - \beta_r\,r \quad (16)$$

Gating variable nested equations

$$\alpha_m(V) = 0.1(V+45)/(1-\exp(-(V+45)/10)) \quad (9)$$

$$\beta_m(V) = 4\ \exp(-(V+70)/18) \quad (10)$$

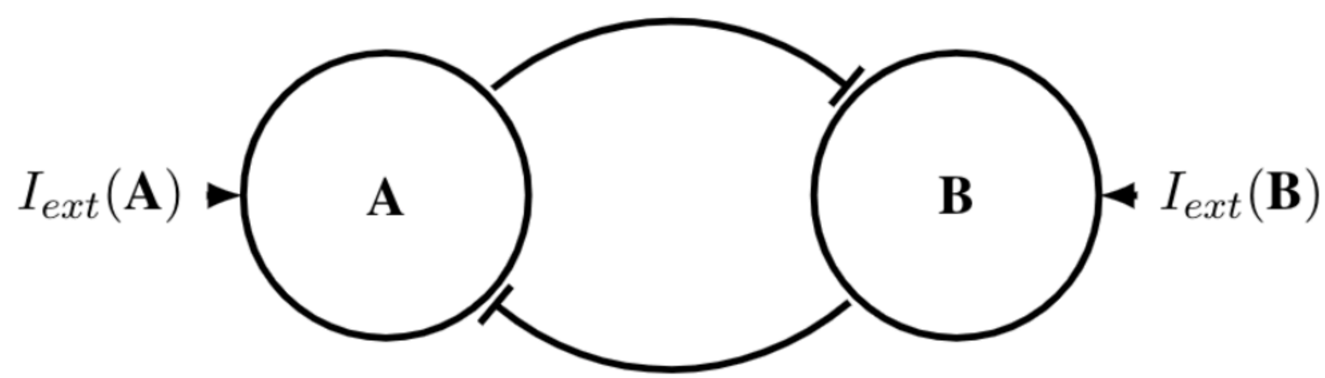
$$\alpha_h(V) = 0.07\ \exp(-(V+70)/20) \quad (11)$$

$$\beta_h(V) = 1/(1+\exp((-(V+40))/10)) \quad (12)$$

$$\alpha_n(V) = 0.01(V+60)/(1-(\exp((-(V+60)/10)-1)) \quad (13)$$

$$\beta_n(V) = 0.125\ \exp(-(V+70)/80) \quad (14)$$

$$[T] = [T]_{max}/(1+\exp(-(V_{pre}-V_p)/K_p)) \quad (17)$$



3. Add Excitation

Current Equations

$$\begin{aligned}
\frac{dV}{dt} &= \frac{1}{C}\left(-I_{Na} - I_K - I_L + I_{ext}\right) && (1)\\
I_{Na} &= g_{Na}\, m^3 h\, (V - E_{Na}) && (2)\\
I_K &= g_K\, n^4\, (V - E_K) && (3)\\
I_L &= g_L\, (V - E_L) && (4)
\end{aligned}$$

$$I_{syn} = g_{GABA_A}\, r\, (V_{post} - E_{Cl}) \qquad (15)$$

Parameters

$$\begin{aligned}
C &= 1\,\mu F/cm^2 & & \\
E_{Na} &= 45\,mV; & g_{Na} &= 120\,mS/cm^2\\
E_K &= -82\,mV; & g_K &= 36\,mS/cm^2\\
E_L &= -59.387\,mV; & g_L &= 0.3\,mS/cm^2
\end{aligned} \qquad (5)$$

$$\begin{aligned}
E_{Cl} &= -80\,mV & &\\
\alpha_r &= 5\,mM^{-1}ms^{-1}; & \beta_r &= 0.18\,ms^{-1}\\
[T]_{max} &= 1.5\,mM & &\\
K_p &= 5\,mV; & V_p &= 7\,mV
\end{aligned} \qquad (18)$$

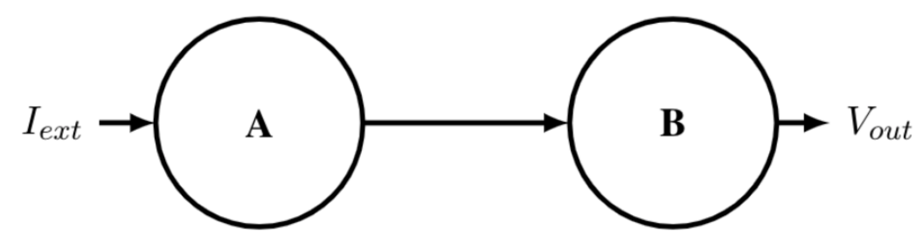
$$\begin{aligned}
E &= -38\,mV & &\\
\alpha_r &= 2.4\,mM^{-1}ms^{-1}; & \beta_r &= 0.56\,ms^{-1}\\
[T]_{max} &= 1.0\,mM & &\\
g_{Glu} &= 0 \text{ to } 0.5\,mS/cm^2 & &
\end{aligned} \qquad (19)$$

Gating variable differential equations

$$\begin{aligned}
\frac{dm}{dt} &= \alpha_m(V)\,(1-m) - \beta_m(V)\,m && (6)\\
\frac{dh}{dt} &= \alpha_h(V)\,(1-h) - \beta_h(V)\,h && (7)\\
\frac{dn}{dt} &= \alpha_n(V)\,(1-n) - \beta_n(V)\,n && (8)\\
\frac{dr}{dt} &= \alpha_r[T]\,(1-r) - \beta_r\,r && (16)
\end{aligned}$$

Gating variable nested equations

$$\begin{aligned}
\alpha_m(V) &= 0.1(V+45)/(1-\exp(-(V+45)/10)) && (9)\\
\beta_m(V) &= 4\,\exp(-(V+70)/18) && (10)\\
\alpha_h(V) &= 0.07\,\exp(-(V+70)/20) && (11)\\
\beta_h(V) &= 1/(1+\exp((-(V+40))/10)) && (12)\\
\alpha_n(V) &= 0.01(V+60)/(1-(\exp((-(V+60)/10)-1)) && (13)\\
\beta_n(V) &= 0.125\,\exp(-(V+70)/80) && (14)\\
[T] &= [T]_{max}/(1+\exp(-(V_{pre}-V_p)/K_p)) && (17)
\end{aligned}$$



Single Neuron

```
def d_single(hh_vars, t, I_ext, g_GABA, g_Glu)
```

Vectorize Variables:

$$\text{hh\_vars} = \begin{bmatrix} V \\ m \\ h \\ n \\ r\_inhibit \\ r\_excite \end{bmatrix}$$

I_ext is applied external current

g_GABA is a matrix of inhibitory synapses where <u>each row is the presynaptic cell and the column is the postsynaptic cell</u>. Same for **g_Glu** except that it is excitatory.

$$\text{g\_GABA} = \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

No connection

$$\text{g\_GABA} = \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

0.1 conductance

```
return system of odes for hh_vars
```

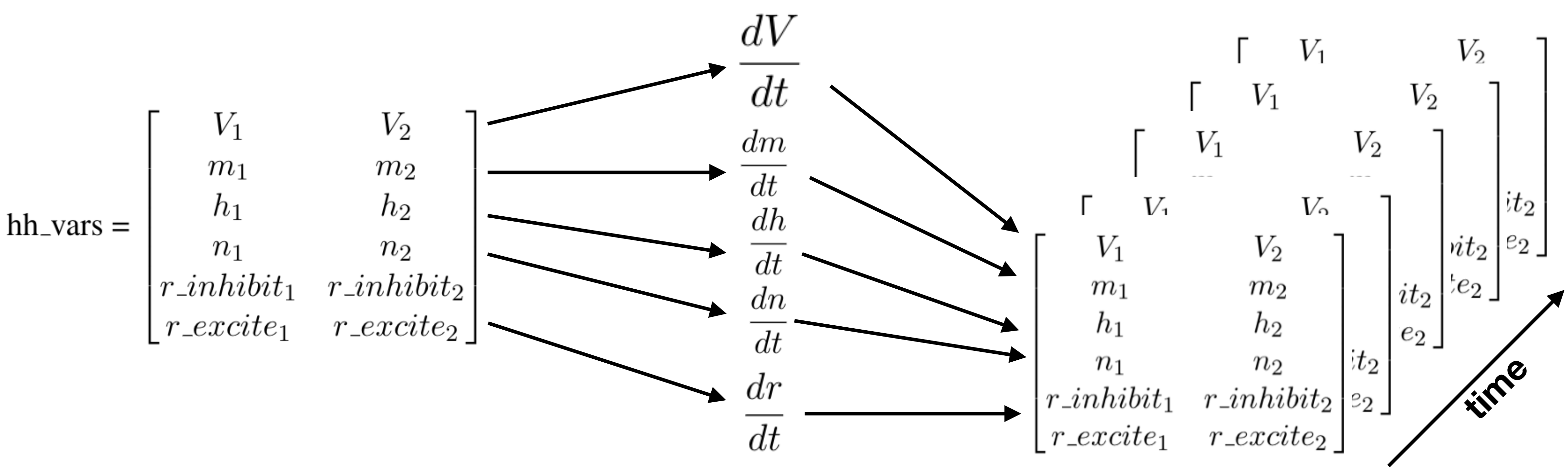
Multiple Neurons

```
def network(I_exts, g_GABA, g_Glu)
```

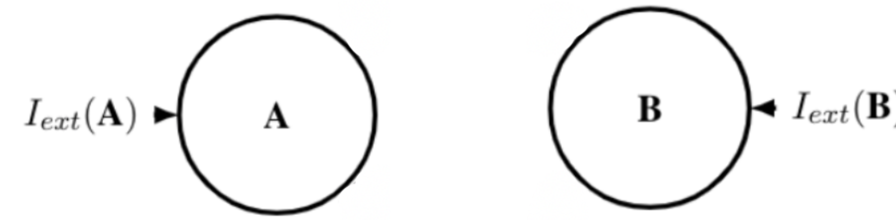
I_exts is a function handle that accepts the time and returns a row vector of current applied to each cell.

Network function solves system of odes for all neuron in network

There is an addition function d in the supplied code that is required to reshape the system of odes to solve for a network



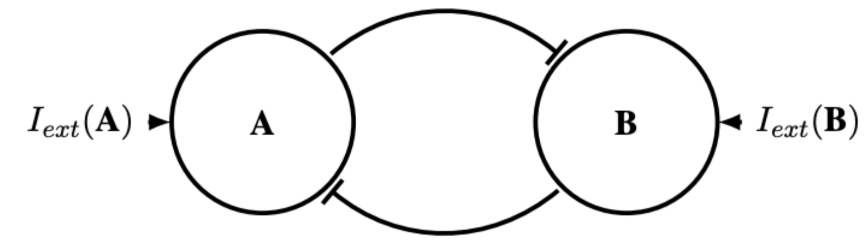
Problem 1




```
# One neuron gets 10 uA (starting at 0ms) and the other gets 20 uA
I_exts_ = sp.array([10, 20])        # uA/cm^2
def I_exts(t): return I_exts_       # Creates time series out of input current, I


# g_GABA and g_Glu matrices are all 0 for no connections
nocon2 = sp.zeros((2,2))
hh_vars = network(I_exts, nocon2, nocon2)


V = hh_vars[0,:,:]


# Looking at spike rate using isi function
for i in range(len(I_exts_)):
    mean_isi, stddev_isi = isi(t[2501:], V[i,2500:]) # skip the first 250ms
```

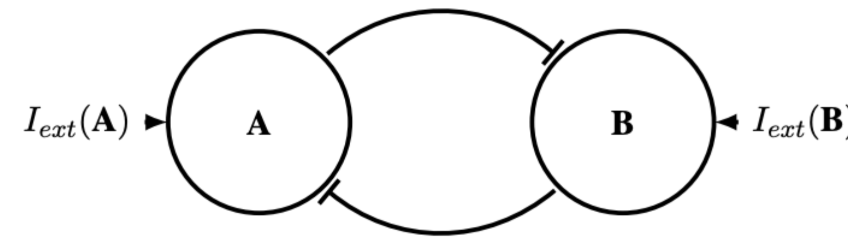
Note: isi function gives interspike interval. 1000 / mean interspike interval converts ms to Hz for frequency

Problem 2




```
# One neuron gets 10 uA (starting at 0ms) and the other gets 20 uA (same input)
I_exts_ = sp.array([10, 20]) # uA/cm^2
def I_exts(t): return I_exts_


# We will test the following g_GABA values:
g_GABAs = np.arange(0, 0.6, 0.1)
g_GABAs = np.append(g_GABAs, np.arange(1.0, 4.0, 0.5))


m = sp.zeros((2, len(g_GABAs))) # initialize means

for i in range(len(g_GABAs)):
    # g_GABA has reciprical connections [[0, x], [x, 0]], g_Glu is all 0 (just inhibitory)
    hh_vars = network(I_exts, sp.array([[0,g_GABAs[i]],[g_GABAs[i],0]]), nocon2)

    # Look at voltage results
    V = hh_vars[0,:,:]

    for j in range(len(I_exts_)):
        m[j,i], s = isi(t[2501:], V[j,2500:]) # skip the first 250ms
        sr = 1000/m[j,i]
        if (sp.isnan(sr)): sr = 0
```

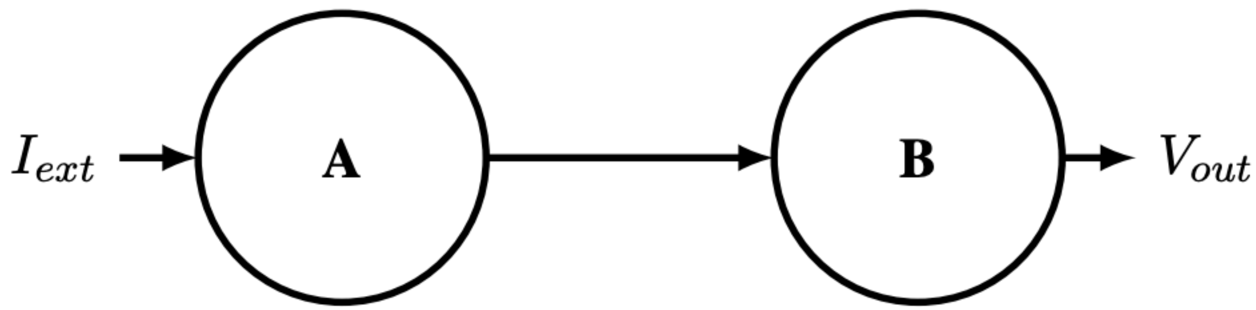
Problem 3



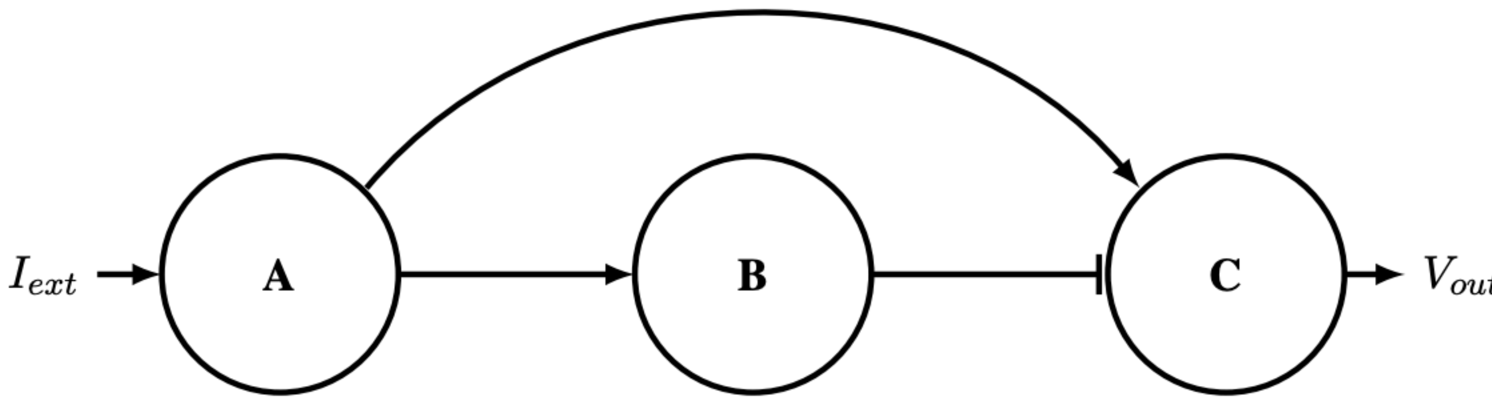

```
# One neuron gets 10 uA (starting at 0ms) and the other gets 10.1 uA (DIFFERENT input)
I_exts_ = sp.array([10, 20]) # uA/cm^2
def I_exts(t): return I_exts_

# g_GABA has reciprical connections [[0, 0.2], [0.2, 0]], g_Glu is all 0
g_GABA = sp.array([[0.0, 0.2],[0.2, 0.0]])

# The beta_r's to test (was previously just 0.18)
beta_rs = np.arange(0.5, 0, -0.1)

p = sp.zeros(len(beta_rs))      # initialize phase array


for i in range(len(beta_rs)):
   beta_r_inhibit = beta_rs[i]     # change beta values in network

    # Run simulation for each beta_r
    hh_vars = network(I_exts, g_GABA, nocon2)

    # Look at voltage results
    V = hh_vars[0,:,:]

    # Find spike phase with python tools
    p[i], mean_isi = spk_phase(t[2501:], V[0, 2500:], V[1, 2500:]) # skip first 250ms
```

Note: remember to set beta_r_inhibit back to its original value of 0.18 when finished with this problem before continuing

Going Forward



g_GABA → g_Glu



g_GABA

g_Glu

What dimensions should these be?