GRADIENT FLOW BEARING ESTIMATION WITH BLIND IDENTIFICATION OF NON-STATIONARY SIGNAL AND INTERFERENCE

Milutin Stanacevic, Gert Cauwenberghs

ECE Department, Johns Hopkins University
Baltimore, MD 21218, USA
E-mail: {*miki,gert*}*@jhu.edu*

Larry Riddle

Signal Systems Corporation
Severna Park, MD 21146, USA
E-mail: *lriddle@signalsystemscorp.com*

ABSTRACT

We present a technique for reliably estimating the 3-D direction cosines of a broadband traveling wave impinging on an array of four sensors of dimensions smaller than the shortest wavelength in the source. Gradient flow converts observed mixtures of delayed source signals into instantaneous linear mixtures of their temporal derivatives through observation of spatial gradients of the field. This formulation is equivalent to independent component analysis (ICA), where the mixing matrix directly yields the direction cosines. Experiments with acoustic data from a microphone array show improved bearing accuracy through second order blind identification (SOBI) of non-stationary noise from interfering sources, along with the signal.

1. INTRODUCTION

Formulating source localization algorithms that perform robustly with sub-wavelength dimensions of the sensor array is a challenging problem introduced by miniaturization of integrated sensors. It is well known that the precision of delay-based bearing estimation degrades with shrinking dimensions (aperture) of the sensor array [1]. Classical time-difference of arrival estimation techniques [2] based on cross-correlation of the signals require high oversampling ratios for estimating small time delays.

Gradient flow [3] avoids the problem of estimating small time delays between sensor observations by relating amplitudes of spatial and temporal gradients in the signal across the sensor array, or equivalently resolve terms in a Taylor expansion of the field [4]. The idea of wavefront sensing in space for localizing sound was first introduced by Blumlein in the 1930s [5].

Section 2 presents the gradient flow approach for blind separation and localization of multiple sources. In Section 3, we show that the problem of estimating time-delays simplifies to least-square problem in the case of one source. Section 4 describes second-order blind identification (SOBI) algorithm [6, 7, 8] used for localization in the case of multiple source and how the assumptions of the algorithm are met in the obtained model. In Section 5 we compare the performance of least-mean-square (LMS) and SOBI algorithm for the case of one bandlimited Gaussian source signal.

This work was partly supported by ONR N00014-99-1-0612, ONR/DARPA N00014-00-C-0315 and N00014-00-1-0838.

2. GRADIENT FLOW LOCALIZATION

A traveling wave emitted by a source is observed over a distribution of sensors in space, which here we consider to be discrete but which could be continuous. We define $\tau(\mathbf{r})$ as the time lag between the wavefront at point $\mathbf{r}$ and the wavefront at the center of the array, *i.e.*, the propagation time $\tau(\mathbf{r})$ is referenced to the center of the array.

For an integrated MEMS or VLSI array with dimensions typically smaller than 1 cm, the distance from the source is much larger than the dimensions of the sensor array, and the *far-field* approximation is a sensible approximation. In the far field, the wavefront delay $\tau(\mathbf{r})$ is approximately linear in the projection of $\mathbf{r}$ on the unit vector $\mathbf{u}$ pointing towards the source,

$$\tau(\mathbf{r}) \approx \frac{1}{c}\mathbf{r} \cdot \mathbf{u} \tag{1}$$

where c is the speed of (acoustic or electromagnetic) wave propagation.

Let $x(\mathbf{r}, t)$ be the signal picked up by a sensor at position $\mathbf{r}$. As one special case we will consider a two-dimensional array of sensors, with position coordinates p and q so that $\mathbf{r}_{pq} = p\mathbf{r}_1 + q\mathbf{r}_2$ with orthogonal vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in the sensor plane. In the *far-field* approximation (1), the sensor observations of the source are advanced in time by $\tau_{pq} = p\tau_1 + q\tau_2$, where

$$\begin{aligned} \tau_1 &= \frac{1}{c}\mathbf{r}_1 \cdot \mathbf{u} \\ \tau_2 &= \frac{1}{c}\mathbf{r}_2 \cdot \mathbf{u} \end{aligned} \tag{2}$$

are the inter-time differences (ITD) of source between adjacent sensors on the grid along the p and q place coordinates, respectively. Knowledge of the *angle coordinates* τ_1 and τ_2 uniquely determines, through (2), the direction vector $\mathbf{u}$ along which source impinges the array, in reference to the $\{p, q\}$ plane.

The signal observed at sensor with position coordinates p and q can be expressed as

$$x_{pq}(t) = \sum_{\ell=1}^{\mathcal{L}} s^\ell(t) + \tau_{pq}^\ell \dot{s}^\ell(t) + \tfrac{1}{2}(\tau_{pq}^\ell)^2 \ddot{s}^\ell(t) + \ldots + n_{pq}(t) \quad (3)$$

where $n_{pq}(t)$ represent additive noise in the sensor observations. A gradient flow formulation is obtained by evaluating spatial gradients of x_{pq} along the p and q position coordinates, around the origin $p = q = 0$:

$$\begin{aligned} \xi_{ij}(t) &\equiv \left. \frac{\partial^{i+j}}{\partial^i p \partial^j q} x_{pq}(t) \right|_{p=q=0} \\ &= \sum_\ell (\tau_1^\ell)^i (\tau_2^\ell)^j \, \frac{d^{i+j}}{d^{i+j}t} s^\ell(t) \; + \; \nu_{ij}(t), \end{aligned} \quad (4)$$

where ν_{ij} are the corresponding spatial derivatives of the sensor noise n_{pq} around the center. Taking spatial derivatives ξ_{ij} of order $i + j \leq k$, and differentiating ξ_{ij} to order $k - (i + j)$ in time yields a number of different linear observations in the kth-order time derivatives of the signals s.

As an example, consider the first-order case $k = 1$, corresponding to (3):

$$\begin{aligned} \xi_{00}(t) &= \textstyle\sum_\ell s^\ell(t) + \nu_{00}(t), \\ \xi_{10}(t) &= \textstyle\sum_\ell \tau_1^\ell \dot{s}^\ell(t) + \nu_{10}(t), \\ \xi_{01}(t) &= \textstyle\sum_\ell \tau_2^\ell \dot{s}^\ell(t) + \nu_{01}(t). \end{aligned} \quad (5)$$

Taking the time derivative of ξ_{00}, we thus obtain from the sensors a linear instantaneous mixture of the time-differentiated source signals,

$$\begin{bmatrix} \dot{\xi}_{00} \\ \xi_{10} \\ \xi_{01} \end{bmatrix} \approx \begin{bmatrix} 1 & \cdots & 1 \\ \tau_1^1 & \cdots & \tau_1^{\mathcal{L}} \\ \tau_2^1 & \cdots & \tau_2^{\mathcal{L}} \end{bmatrix} \begin{bmatrix} \dot{s}^1 \\ \vdots \\ \dot{s}^{\mathcal{L}} \end{bmatrix} + \begin{bmatrix} \dot{\nu}_{00} \\ \nu_{10} \\ \nu_{01} \end{bmatrix}, \quad (6)$$

an equation in the standard form $\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{n}$, where $\mathbf{x}$ is given and the mixing matrix $\mathbf{A}$ and sources $\mathbf{s}$ are unknown. Under the assumptions that the source signals are independent, this formulation is equivalent to standard independent component analysis (ICA), and a number of approaches exist for solving this problem [9]. ICA produces, at best, an estimate $\hat{\mathbf{s}}$ that recovers the original sources $\mathbf{s}$ up to arbitrary scaling and permutation. The direction cosines τ_i^ℓ are found from the ICA estimate of $\mathbf{A}$, after first normalizing each column (*i.e.*, , each source estimate) so that the first row of the estimate $\hat{\mathbf{A}}$, like the real $\mathbf{A}$ according to (6), contains all ones. This simple procedure together with (2) yields estimates of the direction vectors $\hat{\mathbf{u}}^\ell$ along with the source estimates $\hat{s}^\ell(t)$, which are obtained by integrating the components of $\hat{\mathbf{s}}$ over time and removing the DC components.

The proposed gradient flow technique requires computation of temporal derivative and first-order spatial gradients along p and q directions of the signal impinging on the sensor array. Estimates of ξ_{00}, ξ_{10} and ξ_{01} are obtained by finite-difference gradient approximation on a grid (precise up to terms at least of third order), using a planar array of four sensors, illustrated in Figure 1:

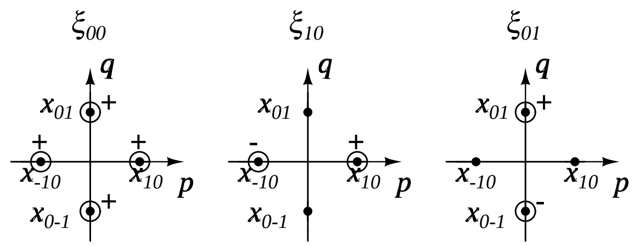

Fig. 1. *Geometry of a planar four sensor array*

$$\begin{aligned} \xi_{00} &\approx \tfrac{1}{4}\big(x_{-1,0} + x_{1,0} + x_{0,-1} + x_{0,1}\big) \\ \xi_{10} &\approx \tfrac{1}{2}\big(x_{1,0} - x_{-1,0}\big) \\ \xi_{01} &\approx \tfrac{1}{2}\big(x_{0,1} - x_{0,-1}\big) \end{aligned} \quad (7)$$

3. SINGLE SOURCE LOCALIZATION

In the case of one directional source, the equation (6) simplifies to

$$\begin{aligned} \dot{\xi}_{00}(t) &= \dot{s}(t) + \dot{\nu}_{00}(t), \\ \xi_{10}(t) &= \tau_1 \dot{s}(t) + \nu_{10}(t), \\ \xi_{01}(t) &= \tau_2 \dot{s}(t) + \nu_{01}(t), \end{aligned} \quad (8)$$

and the problem of estimating time delays converts to standard least-square problem in the unknown delays τ_1 and τ_2, with estimates

$$\begin{aligned} \hat{\tau}_1 &= \frac{\mathrm{E}[\dot{\xi}_{00}\xi_{10}]}{\mathrm{E}[\dot{\xi}_{00}^2]} \\ &= \frac{\mathrm{r}_{\dot{\xi}_{00}\xi_{10}}(0)}{\mathrm{r}_{\dot{\xi}_{00}\dot{\xi}_{00}}(0)} \\ \hat{\tau}_2 &= \frac{\mathrm{E}[\dot{\xi}_{00}\xi_{01}]}{\mathrm{E}[\dot{\xi}_{00}^2]} \\ &= \frac{\mathrm{r}_{\dot{\xi}_{00}\xi_{01}}(0)}{\mathrm{r}_{\dot{\xi}_{00}\dot{\xi}_{00}}(0)}. \end{aligned} \quad (9)$$

From least-square estimates of the time delays, we can directly obtain estimates of azimuth angle θ and elevation angle ϕ angle according to (2):

$$\begin{aligned} \tau_1 &= \frac{1}{c}|\mathbf{r}_1| \; \sin\theta \sin\phi \\ \tau_2 &= \frac{1}{c}|\mathbf{r}_2| \; \cos\theta \sin\phi \end{aligned} \quad (10)$$

An interesting observation is that the estimate of azimuth angle is independent of the speed of sound as it involves spatial gradients only; estimation of the elevation angle on the other hand requires knowledge of the speed of sound in relating spatial and temporal derivatives. The estimate of azimuth angle can be obtained simply from the ratio of delay estimates $\hat{\tau}_1$ and $\hat{\tau}_2$:

$$\hat{\theta} = \arctan \frac{\hat{\tau}_1}{\hat{\tau}_2} \tag{11}$$

or by finding the null of the expression

$$\hat{\tau}_1 \cos(\theta) - \hat{\tau}_2 \sin(\theta) \ . \tag{12}$$

4. MULTIPLE SOURCE LOCALIZATION

The ICA algorithm we have chosen to implement for separation and localization of multiple sources is second-order blind identification (SOBI) algorithm [6, 7, 8]. SOBI deals effectively with non-stationary and even Gaussian statistics for sources and noise in the ICA model (6). The unknown sources are temporal derivatives of impinging signals, leading to temporal structural information of sources that have to be separated. The noise signals in the ICA model can be expanded to sensor noise term and dispersive ambient noise term. The sensor noise contributions are

$$\begin{aligned} \dot{\nu}_{e00} &= \tfrac{1}{4}\big(\dot{e}_{-10} + \dot{e}_{10} + \dot{e}_{0-1} + \dot{e}_{01}\big) \\ \nu_{e10} &= \tfrac{1}{2}\big(e_{10} - e_{-10}\big) \\ \nu_{e01} &= \tfrac{1}{2}\big(e_{01} - e_{0-1}\big) \end{aligned} \tag{13}$$

where e_{10}, e_{-10}, e_{01} and e_{0-1} represent sensor noise at corresponding sensors. Since the cross-correlation of the signal and its derivative is zero and under the assumption that sensors noise is uncorrelated across the sensor array, the sensor noise contribution in ICA model becomes spatially white. The disperse noise covariance matrix, under the assumption that correlation between signals coming from different directions is zero, also becomes diagonal, leading to diagonal covariance matrix of complete noise in observations.

The SOBI algorithm is based on a joint diagonalization of a set of covariance matrices obtained at different time lags. The covariance matrix of observation signals at time lag τ is

$$\mathrm{R}_x(\tau) = \mathrm{E}[\mathbf{x}(\mathrm{t}+\tau)\mathbf{x}^{\mathrm{T}}(\mathrm{t})] = \mathbf{A}\mathrm{R}_{\mathrm{s}}(\tau)\mathbf{A}^{\mathrm{T}} + \sigma(\tau)^2\mathrm{I}, \tag{14}$$

where we used the assumption that noise term is spatially white. After estimating the covariance matrices at different time lags and subtracting the estimated noise contributions, by jointly diagonalizing the obtained set of matrices the mixing matrix $\mathbf{A}$ is estimated. The time delays have to be chosen in such a way that covariance matrices carry maximally different information.

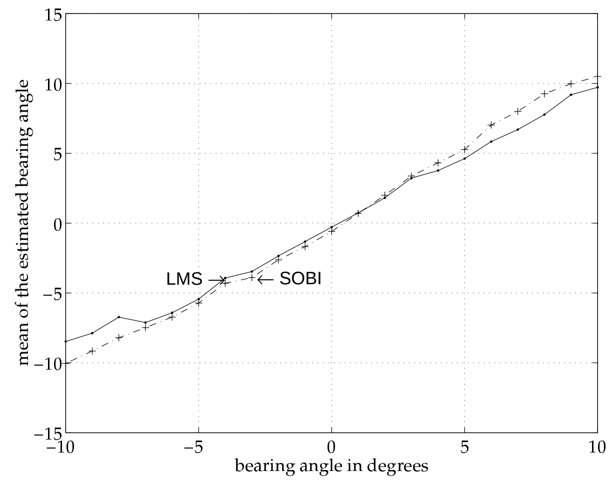

Fig. 2. *Mean value of estimated angle using LMS and SOBI as bearing angle is swept from from -10° to 10°.*

The use of only second-order statistics makes the algorithm more robust than higher-order statistics ICA algorithms. It also allows separation of Gaussian sources. By observing the equation (9), we can notice that LMS solution represents a special case of SOBI, as only a covariance matrix of zero time lag is used for bearing estimation.

5. EXPERIMENTAL SETUP AND RESULTS

To quantify the performance of gradient flow bearing estimation, the experimental setup with one directional source in open-field environment was used. The effective distance between microphones in the planar array of four sensors was 15.87 cm. The sound source was bandlimited (20-300Hz) Gaussian signal presented through a loudspeaker. Data was sampled at 2048 samples per seconds. The distance between loudspeaker and microphone array was approximately 18 m. Signal-to-noise (SNR) ratio was around 25-30 dB. The experiments were performed for bearing angles from -10° to 10° in increments of 1°. The data was played for 30 seconds and the bearing estimates were obtained for 1 second data.

For a localization of a single source, simple expressions can be obtained for the Cramer-Rao lower bound on the variance of bearing angle, assuming Gaussian univariate distributions for the source and noise components [10]. In this experimental setup, the Cramer-Rao bound was around 1 degree. The assumption of uncorrelated noise is violated for subwavelength sensor geometries, and gradient flow exploits correlated noise and temporal dependencies to obtain superior bearing accuracies.

Before bearing estimation of direction cosines using temporal and spatial gradients, common mode offset correction is performed on the estimated spatial gradients. Common mode offsets arise from gain mismatch errors in the

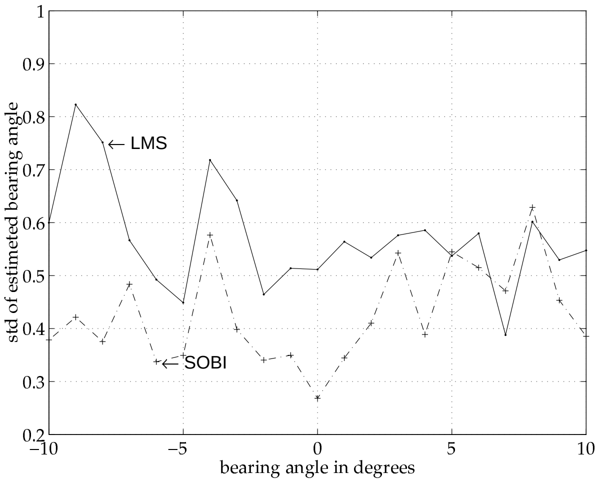

Fig. 3. *Standard deviation of estimated angle using LMS and SOBI as bearing angle is swept from from* -10^o *to* 10^o.

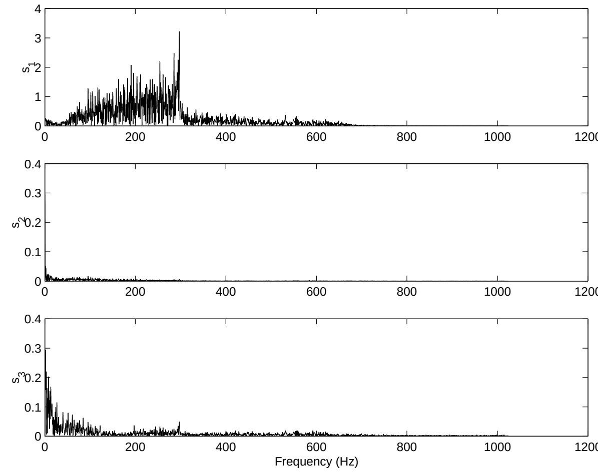

Fig. 4. *Spectrum of the estimated sources obtained by the SOBI algorithm for one second of data and one bearing angle.*

sensors. Since the correlation between any signal and its time-derivative is zero, the correlation between common-mode and gradient variable is also zero. Therefore, using only second-order statistics, we can estimate the leakage of common-mode component in gradient estimates and compensate for it.

The estimates of bearing angle were obtained with both LMS and SOBI. The mean of estimators for bearing angles from -10^o to 10^o is shown in Figure 2, and the standard deviation is shown in Figure 3. As expected, the estimators obtained using ICA algorithm have smaller bias error and variance achieving sub-degree accuracy.

In Figure 4 we show the frequency characteristics of the estimated source signals obtained with SOBI for one second of data and one bearing angle. Since we have three observations in our ICA model, we can estimate up to three sources. The first estimated source is the temporal derivative of the bandlimited (20-300Hz) Gaussian signal presented through the loudspeaker, while the second and third, much smaller in amplitude, represent some interfering background directional sources, or wind noise.

6. CONCLUSION

Gradient flow offers a framework in which ICA can be applied directly to bearing estimation. We obtained improvements in accuracy by modeling signal, noise and interference using second-order temporal decorrelations in the SOBI ICA framework, exploiting non-stationarity in the signal and interfering noise sources. Experimental results demonstrate angular resolutions better than predicted by the Cramer-Rao lower bound for maximum-likelihood estimation assuming stationary uncorrelated Gaussian noise components [10].

7. REFERENCES

[1] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 2nd edition, 1991.

[2] C.H. Knapp and G.C. Carter, "The generalized correlation method for estimation of time delay," *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. 24, no. 4, pp. 320–327, 1976.

[3] G. Cauwenberghs, M. Stanacevic, and G.Zweig, "Blind broadband source localization and separation in miniature sensor arrays," in *Proc. IEEE Int. Symp. Circuits and Systems (ISCAS'2001)*, Sydney, Australia, 2001, vol. 3, pp. 193 –196.

[4] J. Barrère and G. Chabriel, "A compact sensor array for blind separation of sources," *IEEE Trans. Circuits and Systems, Part I*, vol. 49, no. 5, pp. 565–574, 2001.

[5] A.D. Blumlein, "Improvements in and relating to sound-transmission, sound-recording and sound-reproducing systems," *British Patent*, , no. 394325, 1933.

[6] A. Belouchrani, K. Abed-Meraim, J. Cardoso, and E. Mouline, "A blind source separation technique using second-order statistics," *IEEE Transactions on Signal Processing*, vol. 45, no. 2, pp. 434–444, 1997.

[7] L. Molgedey and G. Schuster, "Separation of a mixture of independent signals using time delayed correlations," *Physical Review Letters*, vol. 72, no. 23, pp. 3634–3637, 1994.

[8] A. Ziehe and K. Muller, "Tdsep - an efficient algorithm for blind separation using time structure," in *Proc. Int. Conf. on Artificial Neural Networks(ICANN'98)*, Skovde, Sweden, 1998, pp. 675–680, Springer Verlag.

[9] A. Cichocki and S. Amari, *Adaptive Blind Signal and Image Processing: Learning Algorithms and Applications*, Wiley, 2002.

[10] B. Friedlander, "On the cramer-rao bound for time delay and doppler estimation," *IEEE Transactions on Information Theory*, vol. 30, no. 3, pp. 575–580, 1984.