A Second-Order Log-Domain Bandpass Filter for Audio Frequency Applications

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Paper category: **2.1** *Nonlinear circuits and systems*

***Abstract*— Log-domain filters have recently come into the limelight of the VLSI community as an important class of circuits for implementing continuous-time filters in the current domain [4], [3], [2]. Some papers have discussed log-domain circuit analysis [1] and some have addressed circuit synthesis [2], [5], [6], [7]. In this paper we describe synthesis of a second-order log-domain bandpass filter, address issues related to low-frequency (audio-frequency) filter design, and show experimental results from a system fabricated in a standard 2 µm BiCMOS technology.**

I. Introduction

Log-domain filters comprise a subclass of circuits having externally linear transfer functions but internally nonlinear components [4]. As the name implies, log-domain filters are specifically those circuits whose internal state is a logarithmic function of the input and output. An important property of log-domain filters is that the equations governing the internal nonlinearity of the system are generally tractable, leading to complete solutions which do not require separate DC and transient analyses.

A better understanding of the underlying principles governing log-domain design is now emerging, revealing what circuits are possible for implementing a given filter transfer function. Translinear circuits are known to be important structures in log-domain circuits [1], [6], [5], [8], with translinear analysis greatly simplifying circuit design. Our method of circuit synthesis is closely related to the analysis method presented in [1] and allows us to synthesize filter circuits without resorting to state-space equations or explicitly using transistor equations (*c.f.* [5]).

For purposes of analysis and synthesis, we will consider all transistors (whether BJT or, equivalently, MOSFET operating in weak inversion) to be ideal devices implementing the simplified exponential function $I_c = e^{V_{be}/V_t}$, or inversely, $V_{be} = V_t \ln(I_c)$, where V_t is the thermal voltage (0.025 V at room temperature). Circuit modifications necessary to deal with some of the nonidealities of real devices is addressed in Section V.

II. Principles of log-domain synthesis

Translinear circuit equations give a simple rule for generating a current which is the product or division of other currents. We would like to have a simple rule to generate a current which is the *time derivative* of some other current in the system, so that we can easily construct filter functions in the Laplace domain. In other words, we would like to directly compute

$$I_{out} \propto \dot{I}_{in}. \quad (1)$$

The property of the derivative of an exponential function is the key to this problem. Applying the derivative to the simplified transistor equation results in a simple expression for the time derivative of the current:

$$I_{out} = e^{(V_{be})/V_t} \quad (2)$$

$$\dot{I}_{out} = \frac{\mathrm{d}}{\mathrm{dt}}\left(e^{(V_b - V_e)/V_t}\right) \quad (3)$$

$$= \frac{1}{V_t} I_{out} \frac{\mathrm{d}}{\mathrm{dt}}(V_b - V_e) \quad (4)$$

$$= \frac{1}{V_t} I_{out} \dot{V}_b. \quad (5)$$

$$(6)$$

By adding a capacitor C to the system, we can generate the voltage derivative in terms of a capacitor current I_C, to yield an equation composed entirely of current terms. Fig. 1 shows such a system, a basic building-block of log-domain filters (*c.f.* [2], Fig. 1, and [1], Fig. 1), in which a constant voltage V_{shift} may be inserted between the capacitor node Z and the transistor base without affecting the equation solution:

$$\dot{I}_{out} = \frac{1}{V_t} I_{out} \frac{I_C}{C}. \quad (7)$$

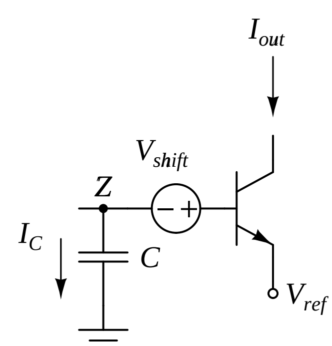

Fig. 1. Filter pole formed using a transconductor.

From this equation we can compute the derivative of a current by multiplying two currents together, which can be done using a translinear loop circuit.

III. CIRCUIT SYNTHESIS

To show how these circuit concepts can be used in practice to synthesize complete filter circuits, consider for instance a generic first-order building-block with the (current) transfer function:

$$\frac{I_{out}}{I_{in}} = \frac{1}{A + \tau s}. \quad (8)$$

We substitute the expression for the current time derivative $\dot{I}_{out}$ for sI_{out}:

$$AI_{out} + \left(\frac{\tau\, I_C}{V_t\, C}\right) I_{out} = I_{in}, \quad (9)$$

$$I_{out}\left(A + \frac{\tau\, I_C}{V_t\, C}\right) = I_{in}. \quad (10)$$

We then can define the time constant τ in terms of some fixed bias voltage I_b:

$$\tau = \frac{V_t\, C}{I_b}. \quad (11)$$

Substitute τ into Equation (10) and multiply through by I_b to get

$$I_{out}\,(AI_b + I_C) = I_{in} I_b. \quad (12)$$

This equation is a four-component translinear loop equation,

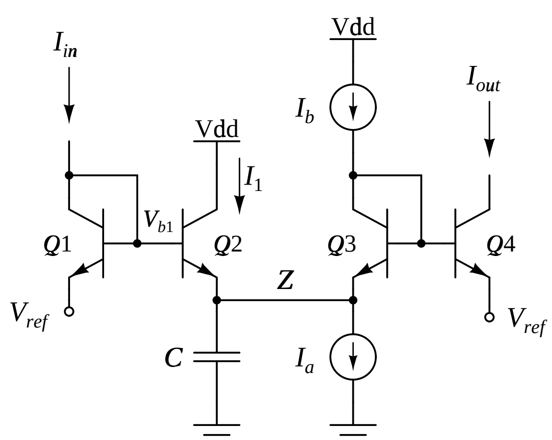

Fig. 2. First-order log-domain building-block.

with the constraint that I_C and I_{out} must have the relationship shown in Fig. 1. One of several possible implementations is shown in Fig. 2, in which transistors $Q1$ through $Q4$ form the translinear loop $I_{out}\, I_1 = I_{in}\, I_b$. $Q3$ is a voltage level-shifter (*c.f.* [2], Fig. 2), and Equation (10) is satisfied if $I_a = I_b\,(1 + A)$. Input and output currents are positively biased to keep the transistors through which they flow operating in the active region, so that the translinear equations remain valid. This DC bias is independent of the input signal.

IV. A SECOND-ORDER BANDPASS FILTER

Generating a higher-order function is a matter of factoring the current transfer function into equations which can directly be implemented by simple subcircuits. For example, consider the second-order bandpass equation

$$\frac{I_{out}}{I_{in}} = \frac{\tau s}{1 + (1/Q)\tau s + \tau^2 s^2}. \quad (13)$$

We can implement this equation directly, yielding a circuit with a bidirectional current output. In general, however, it is more desirable to have the output biased to the same DC level as the input, for example to make it easy to cascade filters together. The most elegant way in terms of circuit implementation is to add a DC component which is a low-pass filtered constant bias current, *i.e.*, the filtering does not affect the constant (the same formula is used in [2] for state-space synthesis of a second-order section):

$$I_{out}(s) = I_{in}(s)\left(\frac{\tau s}{1 + (1/Q)\tau s + \tau^2 s^2}\right) \quad (14)$$

$$+ \; I_{DC}(s)\left(\frac{1}{1 + (1/Q)\tau s + \tau^2 s^2}\right). \quad (15)$$

The reason for adding the complicated second-order low-pass expression becomes apparent when the equation is factored into two simple current transfer functions:

$$\frac{I_{out}}{I_{in} - I_x} = \frac{1}{1/Q + \tau s}; \qquad \frac{I_x}{I_{out} - I_{DC}} = \frac{1}{\tau s}. \quad (16)$$

The right-hand side of both equations is easily implemented as a basic first-order section building block (Fig. 2), the first one made by setting $I_a = I_b(1 + 1/Q)$, and the second one made by setting $I_a = I_b$. The input to each block, however, involves the subtraction of two currents. Because the input to a log-domain filter must be positively DC-biased, this subtraction cannot be performed at the input to the first-order section. Instead, an equivalent value must be subtracted from the central node Z, shown schematically in Fig. 3. If we equate these two circuits, we find that $I'_z = (I_z\, I_b)/I_{out}$. This familiar form can be implemented by a 4-transistor translinear loop circuit, two transistors of which are already in one of the first-order sections. One possible circuit is shown in Fig. 5, where bipolar transistors $Q2$ and $Q7$ through $Q9$ create the translinear loop, but the current I'_{DC} flows into node Y rather than out of it. The current I'_{DC} is mirrored twice, doubled, and subtracted from node Y to make the net value negative.

This circuit is degraded by mismatch in the two mirrors, and it is sensitive to feedback. So the circuit is not appropriate for generating I'_x, which requires good matching and also has feedback from its output back to its input. Instead, we implement the equation $I'_z = (I_z\, I_b)/I_{out}$ another way, as shown in Figs. 4 and 5. Transistors $Q1$ through $Q6$ implement the loop equation $I_x\, {I_b}^2 = I_b\, I'_x\, I_{out}$, which reduces

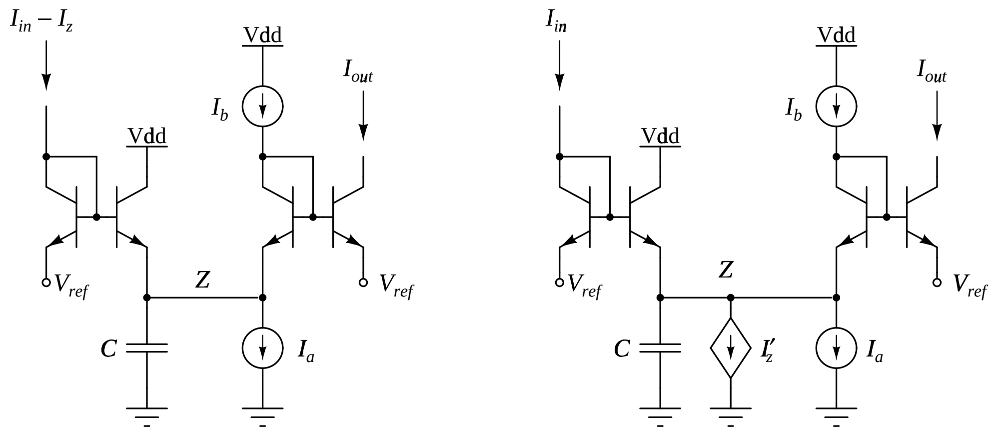

Fig. 3. Computing a current difference at a log-domain filter input. *Left:* The underlying idea, which is physically unrealizable. *Right:* An equivalent working circuit.

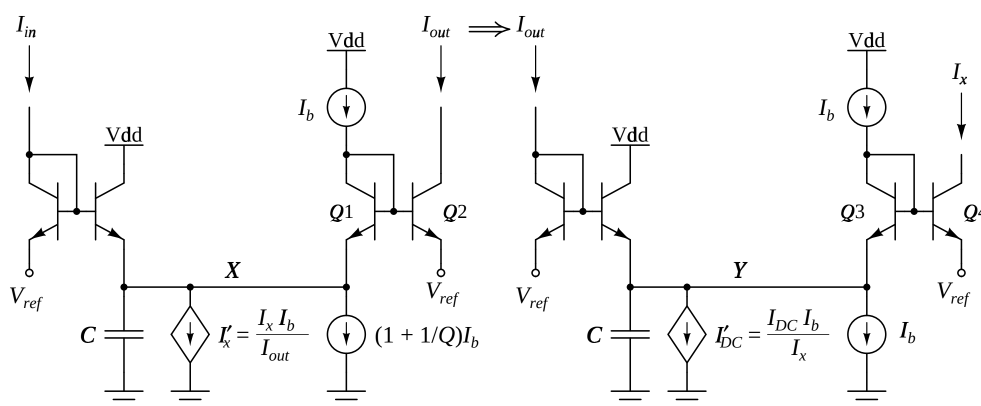

Fig. 4. Bandpass structure formed from first-order sections.

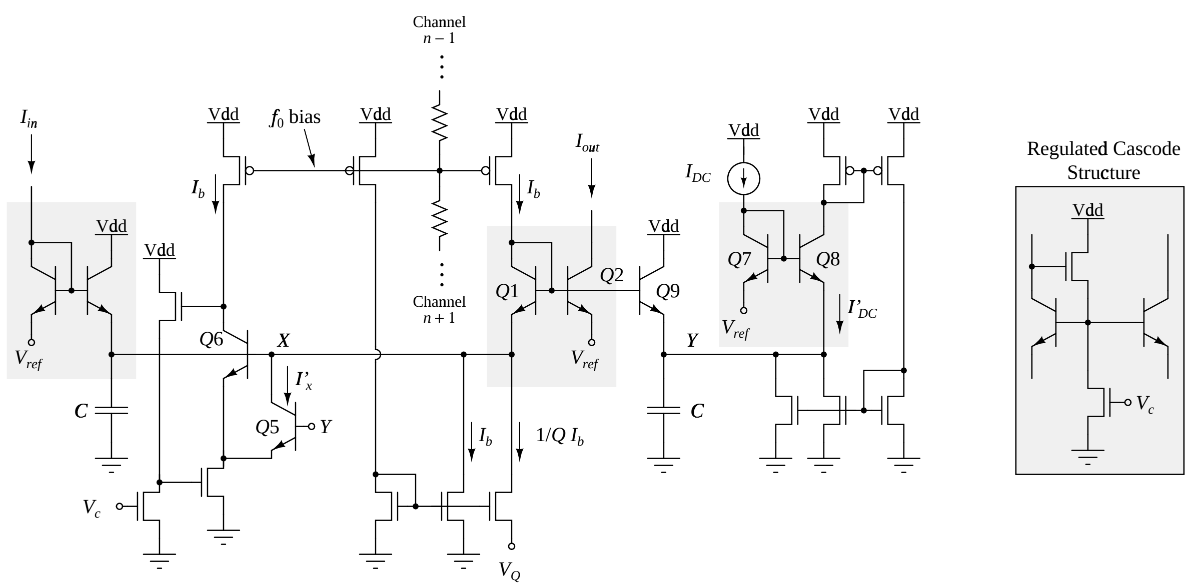

Fig. 5. Complete circuit schematic for the second-order bandpass filter.

to the desired equation. This circuit does not have any unstable states. However, this kind of structure cannot be used to generate I'_{DC} due to current draw through the bipolar bases: therefore both of the methods are required to realize a bandpass filter, each one governed by its particular constraints. Note that in the complete bandpass circuit (Fig. 5), the current I_x is not used directly, so that the output stage ($Q3$ and $Q4$) which generates it is removed. Additionally, the output of the first first-order section is merged with the input of the second, a method which is described briefly in [2].

V. Technology limitations for low-frequency filter design

Previously published work on log-domain filters has generally extolled their potential for high-frequency filter design. Our work instead concentrates on the use of bipolar log-domain filters for audio-frequency applications. The low frequency range requires a large RC time constant, which for the log-domain filters described here is inversely proportional to a bias current. For large-scale integrated systems where capacitors cannot reasonably be made larger than a few picofarads, the bias current can be as low as a few tens of picoamps, which places some important restrictions on circuit technology. Problems arising from established designs are:

1. MOSFET designs have poor matching of currents in mirrors; Input and output are close to the noise floor.
2. Traditional bipolar designs fail due to base current draw and can suffer from β mismatch at low emitter current values.

The first of these two problems is inherent to the design of MOS circuits operating in weak inversion, and is generally difficult to alleviate. The second problem listed stems from the design assumption that base current is negligible in translinear-loop circuits. This assumption becomes invalid at low bias currents, such as when transistor $Q1$ in Fig. 5 attempts to drive $Q2$ when the current I_{out} is many orders of magnitude higher than I_b. This problem has a simple design solution: we have adapted the diode-connected bipolar pairs (shaded regions of Fig. 5) with a "regulated cascode" structure, shown in the shaded box on the right in Fig. 5. V_c is a bias voltage which must be large enough for the circuit to generate the base current drawn by both bipolar transistors. Any remaining dependence on β does not greatly affect circuit performance.

VI. Experimental Results

Our test chip was a complete signal-processing system containing a filterbank of 15 channels, each containing a cascade of two second-order log-domain bandpass filters of the type shown in Fig. 5.

Fig. 6 shows measurements taken from a log-domain bandpass filter at three different frequencies in the audio band and three different Q values. The first filter in the cascade has an adjustible Q value, and the second has a Q fixed at a value less than one and is used primarily to give the response a 40 dB/decade drop on the skirts of the passband. Both filters are biased to have the same peak frequency. Results show that considerable reduction in rolloff occurs on the lower side of the response, due to mismatch of components in the circuit. The circuit behaves as expected near and above the peak frequency, and the response maps well over the entire audio frequency band.

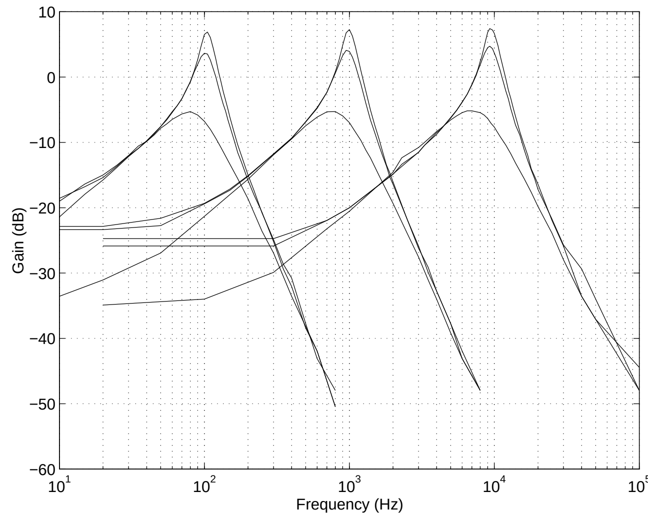

Fig. 6. Measured magnitude response of one bandpass channel in the filterbank system, made of two cascaded second-order log-domain bandpass filters, over three tunings of the center frequency.

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